Logical Properties of Name Restriction

Luca Cardelli
Microsoft Research

Andrew D. Gordon
Microsoft Research

Abstract. We extend the modal logic of ambients described in [7] to the full ambient calculus, including name restriction. We introduce logical operators that can be used to make assertions about restricted names, and we study their properties.

1 Introduction

The π-calculus notion of name restriction [12], initially intended to represent hidden communication channels, has been used also to represent hidden encryption keys [2] and as the basis for definitions of secrecy [2, 4]. In the context of the ambient calculus [6], name restriction can be used to represent hidden locations and (by extrapolating [4] and [5]) secret locations. In general, we would like to have process calculi where we can represent protocols for creating shared encryption keys and secret locations; name restriction seems crucial to all this.

In π-calculus notation, $(\nu n)P$ is a restriction of the name n in the process P, meaning that n is not currently known outside the scope of P. The prefix (νn) is more a bookkeeping device than a barrier. It is quite possible for P to communicate n to some external process; then the restriction (νn) must be formally pushed outwards to encompass the new scope of n and maintain the scoping invariant; this procedure is called *name extrusion*. Processes are considered equivalent up to extrusion; that is, extrusion is not regarded as a computational step. Conversely, when a name is forgotten in part of a process, the scope of (νn) may be restricted; this is called name *intrusion*. Manipulation of (νn) prefixes includes, in particular, renaming and swapping of prefixes, so that there is no obvious way of talking about "the first restricted name" or any particular restricted name of a process.

The ambient calculus can be regarded essentially as an extension of the π-calculus with dynamic location structures. In [7] we present a modal logic for describing properties of ambient calculus processes, with particular emphasis on expressing the structure and evolution of hierarchies of locations. Much of that logic can be applied directly to the π-calculus. However, in [7] we left out name restriction; we now intend to fill that gap in a way that can be applied both to the π-calculus, where names are channels, and to the ambient calculus, where names are locations. In both cases, we need to investigate the logical properties of name restriction.

In our existing logic we can describe detailed properties of processes. If we now consider restriction, what does it mean to describe properties of restricted names? We would like to be able to say, for example, "a shared key is established between locations a and b", or "a secret location is created that only a and b can access". In a protocol that establishes such shared secrets, the secrets are typically represented by restricted names. The problem is that there is no obvious way to talk about such restricted names in the specification of the protocol. We might be tempted to use ordinary existential quantification, and say "there exists a name shared between locations a and b". But this is not good enough, because we want that name to be fresh and unknown to other locations or potential attackers.

Therefore, we want a new form of quantification that can be read as "in the process there exists a restricted name which we shall call x, and such that $\mathcal{A}$", where x is a variable that ranges over names, and $\mathcal{A}$ is some property that may involve x. Let us indicate this quantifier as $(\nu x)\mathcal{A}$; this formula is meant to correspond somehow to a process of the form $(\nu n)P$ where x denotes n. However, since (νn) can float, the matching of (νx) to any particular (νn) is not obvious.

This means that the logical rules of our tentative $(\nu x)\mathcal{A}$ quantifier are going to be fairly complex, or at least unfamiliar. We have approached this complexity by splitting $(\nu x)\mathcal{A}$ into two op-

erators; one for quantifying over fresh names, and one for mentioning restricted names. The first operator is the Gabbay-Pitts quantifier, $!x.\mathcal{A}$, adapted to our context: it quantifies over all names that do not occur free either in the formula $\mathcal{A}$ or in the described process. The second is a binary operator (not a quantifier) called *revelation*, $n\circledR\mathcal{A}$, which means that it is possible to reveal a restricted name as the given name n, and then assert $\mathcal{A}$. (Revelation fails to hold if it would lead to a name clash in the process.)

We investigate the properties of $n\circledR\mathcal{A}$ and $!x.\mathcal{A}$ separately. We combine them to define $(\nu x)\mathcal{A}$ as $!x.x\circledR\mathcal{A}$, and then we study the derived properties of $(\nu x)\mathcal{A}$.

2 Summary of the Ambient Logic

In this section, we provide a quick summary of the ambient calculus. Although this summary is technically self-contained, we assume some knowledge of [6]: see that paper for discussion and motivation. We also summarize the ambient logic studied in [7]. Again, this is self-contained, but knowledge of that paper will help. Two new operators, *revelation* and its adjunct *hiding*, are introduced here, and are discussed in the following sections.

2.1 The Calculus

The syntax of the ambient calculus is defined in the following table:

Processes

P,Q,R ::=	processes	M ::=	capabilities
$(\nu n)P$	restriction	n	name
$\mathbf{0}$	void	*in* M	can enter into M
$P \mid Q$	composition	*out* M	can exit out of M
$!P$	replication	*open* M	can open M
$M[P]$	ambient	ε	null
$M.P$	capability action	$M.M'$	path
$(n).P$	input action		
$\langle M\rangle$	output action		

The set of free names of a process P, written $fn(P)$ is defined as usual, where the only binders are restriction and the input action, so that $fn((\nu n)P) = fn((n).P) = fn(P) - \{n\}$.

We write $P\{n\leftarrow M\}$ for the substitution of the capability M for each free occurrence of the name n in the process P. Similarly for $M\{n\leftarrow M'\}$. We identify processes up to renaming of bound names; that is, we assume, for $m \notin fn(P)$, that $(\nu n)P = (\nu m)P\{n\leftarrow m\}$ and $(n).P = (m).P\{n\leftarrow m\}$.

We use some syntactic conventions. We use parentheses for precedence. The process $\mathbf{0}$ is often omitted in the contexts $n[\mathbf{0}]$ and $M.\mathbf{0}$, yielding $n[]$ and M. Composition has the weakest binding power, so that the expression $(\nu n)P \mid Q$ is read $((\nu n)P) \mid Q$, the expression $!P \mid Q$ is read $(!P) \mid Q$, the expression $M.P \mid Q$ is read $(M.P) \mid Q$, and the expression $(n).P \mid Q$ is read $((n).P) \mid Q$.

Structural congruence is a relation between processes used as an aid in the definition of reduction. With respect to [6], the structural rules for replication have been refined.

The reduction relation describes the dynamic behavior of ambients. In particular, the rules (Red In), (Red Out) and (Red Open) represent mobility, while (Red Comm) represents local communication (see [6] for an extended discussion). For example, the process $a[p[\mathit{out}\ a.\ \mathit{in}\ b.\ \langle m\rangle]] \mid b[\mathit{open}\ p.\ (n).\ n[]]$ represents a packet p that travels out of host a and into host b, where it is opened, and its contents m are read and used to create a new ambient. The process reduces in four steps (illustrating each of the four reduction rules) to the residual process $a[] \mid b[m[]]$.

Structural Congruence

$P \equiv P$	(Struct Refl)	$(\nu n)(\nu m)P \equiv (\nu m)(\nu n)P$	(Struct Res Res)
$P \equiv Q \Rightarrow Q \equiv P$	(Struct Symm)	$(\nu n)\mathbf{0} \equiv \mathbf{0}$	(Struct Res Zero)
$P \equiv Q, Q \equiv R \Rightarrow P \equiv R$	(Struct Trans)	$(\nu n)(P \mid Q) \equiv P \mid (\nu n)Q$ if $n \notin fn(P)$	(Struct Res Par)
		$(\nu n)(m[P]) \equiv m[(\nu n)P]$ if $n \neq m$	(Struct Res Amb)
$P \equiv Q \Rightarrow (\nu n)P \equiv (\nu n)Q$	(Struct Res)		
$P \equiv Q \Rightarrow P \mid R \equiv Q \mid R$	(Struct Par)	$P \mid \mathbf{0} \equiv P$	(Struct Par Zero)
$P \equiv Q \Rightarrow !P \equiv !Q$	(Struct Repl)	$P \mid Q \equiv Q \mid P$	(Struct Par Comm)
$P \equiv Q \Rightarrow n[P] \equiv n[Q]$	(Struct Amb)	$(P \mid Q) \mid R \equiv P \mid (Q \mid R)$	(Struct Par Assoc)
$P \equiv Q \Rightarrow M.P \equiv M.Q$	(Struct Action)		
$P \equiv Q \Rightarrow (n).P \equiv (n).Q$	(Struct Input)	$!\mathbf{0} \equiv \mathbf{0}$	(Struct Repl Zero)
		$!(P \mid Q) \equiv !P \mid !Q$	(Struct Repl Par)
$\varepsilon.P \equiv P$	(Struct ε)	$!P \equiv P \mid !P$	(Struct Repl Copy)
$(M.M').P \equiv M.M'.P$	(Struct .)	$!P \equiv !!P$	(Struct Repl Repl)

Reduction

$n[in\ m.\ P \mid Q] \mid m[R] \longrightarrow m[n[P \mid Q] \mid R]$	(Red In)
$m[n[out\ m.\ P \mid Q] \mid R] \longrightarrow n[P \mid Q] \mid m[R]$	(Red Out)
$open\ n.\ P \mid n[Q] \longrightarrow P \mid Q$	(Red Open)
$(n).P \mid \langle M \rangle \longrightarrow P\{n \leftarrow M\}$	(Red Comm)
$P \longrightarrow Q \Rightarrow (\nu n)P \longrightarrow (\nu n)Q$	(Red Res)
$P \longrightarrow Q \Rightarrow P \mid R \longrightarrow Q \mid R$	(Red Par)
$P \longrightarrow Q \Rightarrow n[P] \longrightarrow n[Q]$	(Red Amb)
$P' \equiv P, P \longrightarrow Q, Q \equiv Q' \Rightarrow P' \longrightarrow Q'$	(Red ≡)
$\longrightarrow^*$	reflexive and transitive closure of $\longrightarrow$

2.2 The Logic

The syntax of logical formulas is summarized below. This is a modal predicate logic with classical negation. As usual, many standard connectives are interdefinable; we take **T**, ¬, ∨, ◇, ∀ as primitive, and **F**, ⇒, ∧, □, ∃ as derived.

Logical Formulas

η	a name n or a variable x		
$\mathcal{A}, \mathcal{B}, \mathcal{C}$ ::=	formulas	$\eta[\mathcal{A}]$	location
T	true	$\mathcal{A}@\eta$	location adjunct
$\neg\mathcal{A}$	negation	$\eta \circledR \mathcal{A}$	revelation
$\mathcal{A} \vee \mathcal{B}$	disjunction	$\mathcal{A} \oslash \eta$	revelation adjunct
0	void	$\Diamond\mathcal{A}$	sometime modality
$\mathcal{A} \mid \mathcal{B}$	composition	$\diamondsuit\mathcal{A}$	somewhere modality
$\mathcal{A} \triangleright \mathcal{B}$	composition adjunct	$\forall x.\mathcal{A}$	universal quantification

The meaning of the formulas will be given shortly in terms of a satisfaction relation. Informally, the first three formulas (true, negation, disjunction) give propositional logic. The next five (void, composition and its adjunct, location and its adjunct) describe tree-like structures of locations. Revelation and its adjunct are new to this paper, and are discussed in detail later. The two spatial and temporal modalities make assertions about states that may happen "further away" in space or time respectively. Quantified variables range only over names: these variables may ap-

pear in the location and revelation constructs, and their adjuncts.

The collections of free names, $fn(\mathcal{A})$, and free variables, $fv(\mathcal{A})$, of a formula $\mathcal{A}$ are defined along standard lines, keeping in mind that there are no name-binding constructs and just one variable-binding construct $(\forall x.\mathcal{A})$.

A formula $\mathcal{A}$ is closed if $fv(\mathcal{A}) = \emptyset$. Substitution $\mathcal{A}\{\eta\leftarrow\mu\}$ of a name or variable μ for another name or variable η in a formula $\mathcal{A}$, is defined in the usual way. We identify formulas up to renaming of bound variables, that is, we assume the identity $\forall x.\mathcal{A} = \forall y.\mathcal{A}\{x\leftarrow y\}$, where $y \notin fv(\mathcal{A})$. We often write $\eta[]$ for $\eta[\mathbf{0}]$, $\mathcal{A}^{\mathbf{F}}$ for $\mathcal{A}\triangleright\mathbf{F}$, and $\mathcal{A}^{\neg}$ for $\neg\mathcal{A}$.

2.3 Satisfaction

The satisfaction relation $P \models \mathcal{A}$ means that the process P satisfies the closed formula $\mathcal{A}$. The definition of satisfaction is based heavily on the structural congruence relation. The satisfaction relation is defined inductively in the following tables, where Π is the sort of processes, Φ is the sort of formulas, ϑ is the sort of variables, and Λ is the sort of names. We use similar syntax for logical connectives at the meta-level and object-level, but this is unambiguous.

The meaning of the temporal modality is given by reductions in the operational semantics of the ambient calculus. For the spatial modality, we need the following definitions. The relation $P{\downarrow}P'$ indicates that P contains P' within exactly one level of nesting. Then, $P{\downarrow}^*P'$ is the reflexive and transitive closure of the previous relation, indicating that P contains P' at some nesting level. Note that P' constitutes the entire contents of an enclosed ambient.

$$P{\downarrow}P' \quad \text{iff} \quad \exists n, P''.\ P \equiv n[P'] \mid P''$$

$\downarrow^*$ is the reflexive and transitive closure of $\downarrow$

Satisfaction

$\forall P\in\Pi.$	$P \models \mathbf{T}$		
$\forall P\in\Pi, \mathcal{A}\in\Phi.$	$P \models \neg\mathcal{A}$	$\triangleq$	$\neg\, P \models \mathcal{A}$
$\forall P\in\Pi, \mathcal{A},\mathcal{B}\in\Phi.$	$P \models \mathcal{A}\vee\mathcal{B}$	$\triangleq$	$P \models \mathcal{A} \vee P \models \mathcal{B}$
$\forall P\in\Pi.$	$P \models \mathbf{0}$	$\triangleq$	$P \equiv \mathbf{0}$
$\forall P\in\Pi, \mathcal{A},\mathcal{B}\in\Phi.$	$P \models \mathcal{A} \mid \mathcal{B}$	$\triangleq$	$\exists P',P''\in\Pi.\ P \equiv P'\mid P'' \wedge P' \models \mathcal{A} \wedge P'' \models \mathcal{B}$
$\forall P\in\Pi, \mathcal{A},\mathcal{B}\in\Phi.$	$P \models \mathcal{A}\triangleright\mathcal{B}$	$\triangleq$	$\forall P'\in\Pi.\ P' \models \mathcal{A} \Rightarrow P\mid P' \models \mathcal{B}$
$\forall P\in\Pi, n\in\Lambda, \mathcal{A}\in\Phi.$	$P \models n[\mathcal{A}]$	$\triangleq$	$\exists P'\in\Pi.\ P \equiv n[P'] \wedge P' \models \mathcal{A}$
$\forall P\in\Pi, \mathcal{A}\in\Phi.$	$P \models \mathcal{A}@n$	$\triangleq$	$n[P] \models \mathcal{A}$
$\forall P\in\Pi, n\in\Lambda, \mathcal{A}\in\Phi.$	$P \models n\circledR\mathcal{A}$	$\triangleq$	$\exists P'\in\Pi.\ P \equiv (\nu n)P' \wedge P' \models \mathcal{A}$
$\forall P\in\Pi, \mathcal{A}\in\Phi.$	$P \models \mathcal{A}\oslash n$	$\triangleq$	$(\nu n)P \models \mathcal{A}$
$\forall P\in\Pi, \mathcal{A}\in\Phi.$	$P \models \Diamond\mathcal{A}$	$\triangleq$	$\exists P'\in\Pi.\ P{\rightarrow}^*P' \wedge P' \models \mathcal{A}$
$\forall P\in\Pi, \mathcal{A}\in\Phi.$	$P \models \Diamond\!\!\!\!-\mathcal{A}$	$\triangleq$	$\exists P'\in\Pi.\ P{\downarrow}^*P' \wedge P' \models \mathcal{A}$
$\forall P\in\Pi, x\in\vartheta, \mathcal{A}\in\Phi.$	$P \models \forall x.\mathcal{A}$	$\triangleq$	$\forall m\in\Lambda.\ P \models \mathcal{A}\{x\leftarrow m\}$

Again, all these logical connectives are described and discussed in [7], except for revelation and its adjunct, which are the subject of Section 3.

Remark: Given our policy of identifying formulas up to the renaming of bound variables, we need to check that satisfaction is well defined with respect to the equation $\forall x.\mathcal{A} = \forall y.\mathcal{A}\{x\leftarrow y\}$, where $y \notin fv(\mathcal{A})$. We need to show for all processes P, formulas $\mathcal{A}$, and variables x and y such that $y \notin fv(\mathcal{A})$ that $P \models \forall x.\mathcal{A}$ if and only if $P \models \forall y.\mathcal{A}\{x\leftarrow y\}$. By definition, $P \models \forall y.\mathcal{A}\{x\leftarrow y\}$ if and only if $\forall m\in\Lambda.\ P \models \mathcal{A}\{x\leftarrow y\}\{y\leftarrow m\}$. Since $y \notin fv(\mathcal{A})$, we have $\mathcal{A}\{x\leftarrow y\}\{y\leftarrow m\} = \mathcal{A}\{x\leftarrow m\}$. Therefore, $P \models \forall y.\mathcal{A}\{x\leftarrow y\}$ if and only if $\forall m\in\Lambda.\ P \models \mathcal{A}\{x\leftarrow m\}$ This is the definition of satisfaction for $\forall x.\mathcal{A}$. So it follows that $y \notin fv(\mathcal{A})$ implies that $P \models \forall x.\mathcal{A}$ if and only if $P \models \forall y.\mathcal{A}\{x\leftarrow y\}$. □

Fundamental Lemmas

The following lemmas are crucial in what follows.

2-1 Lemma (Satisfaction is up to ≡)

$(P \vDash \mathcal{A} \wedge P \equiv P') \Rightarrow P' \vDash \mathcal{A}$ □

2-2 Lemmas (Inversion)

(1) $P \equiv Q \;\Rightarrow\; fn(P) = fn(Q)$

(2) $(\nu n)P \equiv \mathbf{0} \;\Rightarrow\; P \equiv \mathbf{0}$

(3) $(\nu n)P \equiv m[Q] \;\Rightarrow\; \exists R{\in}\Pi.\ P \equiv m[R] \wedge Q \equiv (\nu n)R$ (for $n \neq m$)

(4) $(\nu n)P \equiv Q' \mid Q'' \;\Rightarrow\; \exists R',R''{\in}\Pi.\ P \equiv R' \mid R'' \wedge Q' \equiv (\nu n)R' \wedge Q'' \equiv (\nu n)R''$ □

Remark. It is not true that $(\nu n)P \equiv (\nu n)Q$ implies $P \equiv Q$. Take $P = n[]$ and $Q = (\nu n)n[]$; then $(\nu n)n[] \equiv (\nu n)(\nu n)n[]$ but $n[] \not\equiv (\nu n)n[]$. □

2-3 Lemma (Fresh renaming preserves ⊨)

For all closed formulas $\mathcal{A}$, processes P, and names m, m',
if $m' \notin fn(P) \cup fn(\mathcal{A})$ then $P \vDash \mathcal{A} \Leftrightarrow P\{m \leftarrow m'\} \vDash \mathcal{A}\{m \leftarrow m'\}$. □

The proof of Lemma 2-1 is an induction on the structure of $\mathcal{A}$. See [8] for Lemmas 2-2. The proof of Lemma 2-3 is by induction on the number of symbols in the closed formula $\mathcal{A}$, which is unchanged by substituting a name for a variable or another name; this proof is an extension of the analogous one from [7] with cases for revelation and hiding. It is common for semantic properties of the π-calculus and its descendants to be preserved by fresh renaming; an early example is a fresh renaming lemma for strong bisimulation in the original article on the π-calculus [13, 9].

2.4 Validity

Valid Formulas, Sequents, and Rules

A closed formula is valid when it is satisfied by all processes. A general formula is valid when it is valid under any closed instantiation of its free variables with names.

More precisely, if $fv(\mathcal{A})=\{x_1, ..., x_k\}$ are the free variables of $\mathcal{A}$ and $\varphi{\in}\vartheta{\rightarrow}\Lambda$ is a substitution of names for variables such that $dom(\varphi){\supseteq}fv(\mathcal{A})$, then we write $\mathcal{A}_\varphi$ for $\mathcal{A}\{x_1{\leftarrow}\varphi(x_1), ..., x_k{\leftarrow}\varphi(x_k)\}$, and we define:

Valid Formulas

$\boldsymbol{vld}(\mathcal{A})_\varphi \;\triangleq\; \forall P{\in}\Pi.\ P \vDash \mathcal{A}_\varphi$ for $\varphi{\in}\vartheta{\rightarrow}\Lambda$ with $dom(\varphi){\supseteq}fv(\mathcal{A})$

$\boldsymbol{vld}(\mathcal{A}) \;\triangleq\; \forall \varphi{\in}fv(\mathcal{A}){\rightarrow}\Lambda.\ \boldsymbol{vld}(\mathcal{A})_\varphi$

We use validity for interpreting logical inference rules, as described in the following tables. We use a linearized notation for inference rules, where the usual horizontal bar separating antecedents from consequents is written '⊦' in-line, and ';' is used to separate antecedents.

Sequents are interpreted as follows. A simple sequent $\mathcal{A} \vdash \mathcal{B}$ is interpreted as the validity of the formula $\mathcal{A}{\Rightarrow}\mathcal{B}$. Sequents with conditions about disjointness of variables, or disjointness of variables from names, are reduced to simple sequents, as described below. Equality of names $\eta{=}\mu$ is definable in the logic as $\eta[\mathbf{T}]@\mu$ [7].

Sequents

$\mathcal{A} \vdash \mathcal{B} \;\triangleq\; \boldsymbol{vld}(\mathcal{A}{\Rightarrow}\mathcal{B})$

$\mathcal{A} \vdash \mathcal{B}\ (\eta_1{\neq}\mu_1, ..., \eta_n{\neq}\mu_n) \;\triangleq\; (\eta_1{\neq}\mu_1 \wedge ... \wedge \eta_n{\neq}\mu_n \wedge \mathcal{A}) \vdash \mathcal{B}$

$\mathcal{A} \dashv\vdash \mathcal{B}\ (\Xi) \;\triangleq\; (\mathcal{A} \vdash \mathcal{B}\ (\Xi)) \wedge (\mathcal{B} \vdash \mathcal{A}\ (\Xi))$ where $\Xi = \eta_1{\neq}\mu_1, ..., \eta_n{\neq}\mu_n$

For example: $\mathcal{A} \vdash \mathcal{B}$ means $\forall \varphi{\in}fv(\mathcal{A}{\Rightarrow}\mathcal{B}){\rightarrow}\Lambda.\ \forall P{\in}\Pi.\ P \vDash \mathcal{A}_\varphi \Rightarrow P \vDash \mathcal{B}_\varphi$.

Logical rules are interpreted as follows, where $\mathcal{S}$ are sequents (any of the three forms above,

including sequents with side conditions and double sequents):

Rules

$\mathcal{S}_1; ...; \mathcal{S}_n \nmid \mathcal{S}_0 \quad \triangleq \quad (\mathcal{S}_1 \wedge ... \wedge \mathcal{S}_n) \Rightarrow \mathcal{S}_0$
$\mathcal{S}_1 \nmid\nmid \mathcal{S}_2 \quad \triangleq \quad \mathcal{S}_1 \nmid \mathcal{S}_2 \quad \wedge \quad \mathcal{S}_2 \nmid \mathcal{S}_1$

The definition of validity for formulas with free variables allows us to handle quantification over names. We obtain the validity of the following standard rules for the universal quantifier, and for the definable existential quantifier:

Quantification

($\forall$ L)	$\mathcal{A}\{x \leftarrow \eta\} \vdash \mathcal{B} \nmid \forall x.\mathcal{A} \vdash \mathcal{B}$	where η is a name or a variable
($\forall$ R)	$\mathcal{A} \vdash \mathcal{B} \nmid \mathcal{A} \vdash \forall x.\mathcal{B}$	where $x \notin fv(\mathcal{A})$
($\exists$ L)	$\mathcal{A} \vdash \mathcal{B} \nmid \exists x.\mathcal{A} \vdash \mathcal{B}$	where $x \notin fv(\mathcal{B})$
($\exists$ R)	$\mathcal{A} \vdash \mathcal{B}\{x \leftarrow \eta\} \nmid \mathcal{A} \vdash \exists x.\mathcal{B}$	where η is a name or a variable

***Remark:* ($\forall$ R).** The distinction between variables and names in formulas, and the use of variables (as opposed to names) in quantification is crucial for ($\forall$ R). The version of ($\forall$ R) with names instead of variables:

$$\mathcal{A} \vdash \mathcal{B} \nmid \mathcal{A} \vdash \forall n.\mathcal{B} \qquad \text{where } n \notin fn(\mathcal{A}),$$

is not sound. Consider the valid sequent $m[\mathbf{T}] \vdash \neg n[\mathbf{T}]$. If quantification binders were names, then the rule ($\forall$ R) could be used to produce $m[\mathbf{T}] \vdash \forall n.\neg n[\mathbf{T}]$, which is not valid. Since quantification binders are variables, one can only deduce $m[\mathbf{T}] \vdash \forall x.\neg n[\mathbf{T}]$. □

***Remark:* ($\forall$ L).** The use of substitutions that admit variables, in addition to names, in ($\forall$ L), is crucial. Otherwise, if ($\forall$ L) is formulated as $\mathcal{A}\{x \leftarrow m\} \vdash \mathcal{B} \nmid \forall x.\mathcal{A} \vdash \mathcal{B}$, there does not seem to be any way to derive, for example:

$$\mathcal{A} \vdash \mathcal{B} \nmid \forall x.\mathcal{A} \vdash \forall x.\mathcal{B}$$

which is obtained by starting from $\mathcal{A}\{x \leftarrow x\} \vdash \mathcal{B}$ and applying ($\forall$ L) and then ($\forall$ R). □

A number of proof principles can be derived from the definition of validity:

Instantiation Principle. Let $\mathcal{S}$ be a one-directional sequent, then, for any x,n:

(Inst) $\mathcal{S} \nmid \mathcal{S}\{x \leftarrow n\}$

Substitution Principle. Let $\mathcal{B}\{-\}$ be a formula with a set of formula holes, indicated by –, and let $\mathcal{B}\{\mathcal{A}\}$ denote the formula obtained by filling those holes with $\mathcal{A}$, after renaming the bound variables of $\mathcal{B}$ so they do not capture free variables of $\mathcal{A}$.

(Subst) $\mathcal{A}' \dashv\vdash \mathcal{A}'' \nmid \mathcal{B}\{\mathcal{A}'\} \dashv\vdash \mathcal{B}\{\mathcal{A}''\}$

Case Analysis Principle. A case analysis principle is useful for proofs involving equality and inequality; inequalities often occur as side-conditions of primitive and derived rules. A predicate $\mathcal{A}$ is *classical* iff $\forall \varphi \in fv(\mathcal{A}) \rightarrow \Lambda.\ \{P \parallel P \models \mathcal{A}_\varphi\} \in \{\Pi, \emptyset\}$. Note that $\mathbf{T}$, $\mathbf{F}$, and $\eta = \mu$ are classical predicates; so is $\mathcal{A}^{\mathbf{F}}$, for any $\mathcal{A}$ (meaning that $\mathcal{A}$ is unsatisfiable), and so is the conjunction, disjunction, and negation of classical predicates. Let $\mathcal{S}\{-\}$ be a one-directional sequent with a set of formula holes, and $\mathcal{A}$ be a classical predicate. Then:

(Case Analysis) $\mathcal{S}\{\mathbf{T}\}; \mathcal{S}\{\mathbf{F}\} \nmid \mathcal{S}\{\mathcal{A}\}$

3 Revelation

We now study the logical connectives $\eta \circledR \mathcal{A}$ (*revelation*), and $\mathcal{A} \oslash \eta$ (*revelation adjunct* or *hiding*). These connectives make assertions about restricted names that occur in processes.

3.1 Satisfaction

The formula $\eta \circledR \mathcal{A}$ is used to *reveal* a restricted name; it is read "reveal η then $\mathcal{A}$", where η is either a name (n) or the occurrence of a variable (x) that denotes a name. A process P satisfies the formula $n \circledR \mathcal{A}$ if it is possible to pull a restricted name occurring in P to the top and rename it n, and then strip off the restriction to leave a residual process that satisfies $\mathcal{A}$.

We cannot rename a top-level restricted name of P to n if n is already free in P. Therefore, a revelation formula provides a way of testing for the free names of the underlying process P, as we discuss below.

The inverse (technically, the adjunct) of revelation is called *hiding*: $\mathcal{A} \oslash \eta$, which is read "hide η then $\mathcal{A}$". A process P satisfies the formula $\mathcal{A} \oslash n$ if $(\nu n)P$ satisfies $\mathcal{A}$, that is, if it is possible to hide n in P and then satisfy $\mathcal{A}$. The satisfaction relation $P \models \mathcal{A}$ for revelation and hiding is repeated below:

Satisfaction for Revelation and Hiding

$$P \models n \circledR \mathcal{A} \;\triangleq\; \exists P' \in \Pi.\ P \equiv (\nu n)P' \wedge P' \models \mathcal{A}$$
$$P \models \mathcal{A} \oslash n \;\triangleq\; (\nu n)P \models \mathcal{A}$$

Here are some simple examples:

$(\nu n)n[] \models n \circledR \mathbf{T}$	because $n[] \models \mathbf{T}$
$\mathbf{0} \models n \circledR \mathbf{T}$	because $\mathbf{0} \equiv (\nu n)\mathbf{0}$ and $\mathbf{0} \models \mathbf{T}$
$\neg\ n[] \models n \circledR \mathbf{T}$	because there is no process $(\nu n)P' \equiv n[]$
$(\nu m)m[] \models n \circledR n[]$	because $(\nu m)m[] \equiv (\nu n)n[]$ and $n[] \models n[]$
$m[] \models (n \circledR n[]) \oslash m$	because $(\nu m)m[] \models n \circledR n[]$

Revelation gives us a way to talk about the free (or "known") names of a process. This can be embodied in a derived operator $\copyright n$, satisfied by a process P iff $n \in fn(P)$. Several other derived connectives can be imagined:

Examples of Derived Connectives

$\copyright\eta$	$\triangleq \neg\eta \circledR \mathbf{T}$	contains η free (it is not possible to reveal η)
closed	$\triangleq \neg\exists x.\copyright x$	no free names
separate	$\triangleq \neg\exists x.(\copyright x \mid \copyright x)$	no shared free names
atmostfree η	$\triangleq$ *closed* $\oslash \eta$	contains at most η free

For clarity, we expand the definitions of these derived connectives:

Expanded Definitions

$\forall P \in \Pi.$	$P \models \copyright n$	iff	$\neg\exists P' \in \Pi.\ P \equiv (\nu n)P'$	iff $n \in fn(P)$
$\forall P \in \Pi.$	$P \models$ *closed*	iff	$\forall n \in \Lambda.\ \exists P' \in \Pi.\ P \equiv (\nu n)P'$	iff $fn(P) = \emptyset$
$\forall P \in \Pi.$	$P \models$ *separate*	iff	$\neg\exists n \in \Lambda.\ \exists P', P'' \in \Pi.\ P \equiv P' \mid P'' \wedge n \in fn(P') \wedge n \in fn(P'')$	
$\forall P \in \Pi.$	$P \models$ *atmostfree* n	iff	$\forall m \in \Lambda.\ \exists P' \in \Pi.\ (\nu n)P \equiv (\nu m)P'$	iff $fn(P) \subseteq \{n\}$

Examples:

$n[] \models \copyright n$	because $\neg\exists P' \in \Pi.\ n[] \equiv (\nu n)P'$
$(\nu m)m[] \models$ *closed*	because $\forall n \in \Lambda.\ (\nu m)m[] \equiv (\nu n)(\nu m)m[]$
$n[] \mid m[] \mid (\nu p)(p[] \mid p[]) \models$ *separate*	

3.2 Rules

Before giving our set of primitive rules of revelation and hiding, we discuss the most interesting

properties of ® and ⦸ that are derived in this section. In order to emphasize some symmetries, we use here a combination of primitive and derived rules.

First, the cancellation and swapping properties of double restriction, $(\nu n)(\nu n)P \equiv (\nu n)P$ and $(\nu n)(\nu m)P \equiv (\nu m)(\nu n)P$, are inherited by both ® and ⦸:

$$n®n®\mathcal{A} \dashv\vdash n®\mathcal{A} \qquad n®m®\mathcal{A} \vdash m®n®\mathcal{A}$$
$$\mathcal{A}⦸n⦸n \dashv\vdash \mathcal{A}⦸n \qquad \mathcal{A}⦸m⦸n \vdash \mathcal{A}⦸n⦸m$$

Next, consider the combinations:

$$n®(\mathcal{A}⦸n) \qquad (n®\mathcal{A})⦸n$$

We see easily that $P \models n®(\mathcal{A}⦸n)$ means that $P \models \mathcal{A}$ and that $n \notin fn(P)$, where $n \notin fn(P)$ can be written also as $P \models n®\mathbf{T}$. Instead, $P \models (n®\mathcal{A})⦸n$ means that, although P may not satisfy $\mathcal{A}$, if we hide n in P we obtain something where we can reveal n and satisfy $\mathcal{A}$. For example, $(\nu m)n[m[]] \not\models m®m[n[]]$, but $(\nu m)n[m[]] \models (n®m®m[n[]])⦸n$, because $(\nu n)(\nu m)n[m[]] \equiv (\nu n)(\nu m)m[n[]] \models n®m®m[n[]]$. In other words, $P \models (n®\mathcal{A})⦸n$ means that we can satisfy $\mathcal{A}$ by hiding the name n of P, and revealing a possibly different restricted name of P as n. We obtain the properties:

$$n®(\mathcal{A}⦸n) \dashv\vdash \mathcal{A} \wedge n®\mathbf{T}$$
$$n®(\mathcal{A}⦸n) \vdash \mathcal{A} \qquad \mathcal{A} \vdash (n®\mathcal{A})⦸n$$
$$n®(\mathcal{A}⦸n) \vdash \mathcal{A}⦸n \qquad \mathcal{A}⦸n \vdash (n®\mathcal{A})⦸n$$
$$n®(\mathcal{A}⦸n) \vdash n®\mathcal{A} \qquad n®\mathcal{A} \vdash (n®\mathcal{A})⦸n$$

The interactions of ® and ⦸ with | are the most interesting, and the most complex. There are basically three distribution rules: distribution of ® over | in both directions (with a constraint), unrestricted distribution of ⦸ over | in one direction, and distribution of $n®((-)⦸n)$ over | in both directions.

$$n®(\mathcal{A} \mid n®\mathcal{B}) \dashv\vdash n®\mathcal{A} \mid n®\mathcal{B}$$
$$(\mathcal{A} \mid \mathcal{B})⦸n \vdash \mathcal{A}⦸n \mid \mathcal{B}⦸n$$
$$n®((\mathcal{A} \mid \mathcal{B})⦸n) \dashv\vdash n®(\mathcal{A}⦸n) \mid n®(\mathcal{B}⦸n)$$

The first rule embodies the scope extrusion rule, $(\nu n)(P \mid Q) \equiv ((\nu n)P) \mid Q$ if $n \notin fn(Q)$. This can be seen more clearly if we note that $n \notin fn(Q)$ is equivalent to $Q \equiv (\nu n)Q$; then the extrusion rule can be written as $(\nu n)(P \mid (\nu n)Q) \equiv ((\nu n)P) \mid ((\nu n)Q)$ with no side condition.

The second rule implies that if $(\nu n)(P \mid Q) \models \mathcal{A} \mid \mathcal{B}$ then it is possible to distribute the restriction so that $(\nu n)P \models \mathcal{A}$ and $(\nu n)Q \models \mathcal{B}$; this is a consequence of Lemma 2-2(4).

The last rule looks mysterious, but has a simple interpretation. According to one of the equivalences above, it can be rewritten as $(\mathcal{A} \mid \mathcal{B}) \wedge n®\mathbf{T} \dashv\vdash (\mathcal{A} \wedge n®\mathbf{T}) \mid (\mathcal{B} \wedge n®\mathbf{T})$; that is, the name n does not occur in a parallel composition iff it does not occur in either component. The right-to-left direction is actually a derivable rule.

A similar set of rules holds for distribution of ® and ⦸ over $n[-]$:

$$n®m[\mathcal{A}] \dashv\vdash m[n®\mathcal{A}] \qquad (n \neq m)$$
$$m[\mathcal{A}]⦸n \dashv\vdash m[\mathcal{A}⦸n] \qquad (n \neq m)$$
$$n[\mathcal{A}]⦸n \dashv\vdash \mathbf{F}$$
$$n®(m[\mathcal{A}]⦸n) \dashv\vdash m[n®(\mathcal{A}⦸n)] \qquad (n \neq m)$$

The distribution of $n®-$ over $m[-]$ (first rule) holds in both directions as long as $n \neq m$.

The distribution of $-⦸n$ over $m[-]$ (second and third rules) comes in two cases, depending on whether $n=m$. In each case, the right-to-left direction is derivable. From $n[\mathbf{T}]⦸n \vdash \mathbf{F}$ we can derive $n®\mathbf{T} \vdash \neg n[\mathbf{T}]$, which means that if a name n does not occur free in a process, the process cannot be a location named n.

The distribution of $n\circledR((-)\oslash n)$ over $m[-]$ (fourth rule) is derivable in both directions, from the first two rules. Again, this rule can be rewritten as $m[\mathcal{A}] \wedge n\circledR\mathbf{T} \dashv\vdash m[\mathcal{A} \wedge n\circledR\mathbf{T}]$ $(n \neq m)$; that is, the name n does not occur in a location iff it is distinct from the name of the location and it does not occur inside the location.

Finally, $\circledR$ and $\oslash$ commute in one direction:

$$m\circledR(\mathcal{A}\oslash n) \vdash (m\circledR\mathcal{A})\oslash n$$

We now take the following set of rules as primitive (i.e., we verify their validity in the model). The first group handles double revelation, distribution of $\circledR$ over $\vee$, congruence of $\circledR$ with $\vdash$, the adjunction rule connecting $\circledR$ and $\oslash$, and the rather curious but very useful fact that $\neg$ commutes with $\oslash$. The other three groups deal with the interactions of $\circledR$ and $\oslash$ with $\mathbf{0}$, $|$, and $n[-]$.

3-1 Proposition (Validity: Revelation Rules)

$(\circledR)$	$x\circledR x\circledR\mathcal{A} \dashv\vdash x\circledR\mathcal{A}$	$(\circledR \mid)$	$x\circledR(\mathcal{A} \mid x\circledR\mathcal{B}) \dashv\vdash x\circledR\mathcal{A} \mid x\circledR\mathcal{B}$
$(\circledR\ \circledR)$	$x\circledR y\circledR\mathcal{A} \vdash y\circledR x\circledR\mathcal{A}$	$(\oslash \mid)$	$(\mathcal{A} \mid \mathcal{B})\oslash x \vdash \mathcal{A}\oslash x \mid \mathcal{B}\oslash x$
$(\circledR \vee)$	$x\circledR(\mathcal{A}\vee\mathcal{B}) \vdash x\circledR\mathcal{A} \vee x\circledR\mathcal{A}$	$(\circledR \oslash \mid)$	$x\circledR((\mathcal{A} \mid \mathcal{B})\oslash x) \vdash x\circledR(\mathcal{A}\oslash x) \mid x\circledR(\mathcal{B}\oslash x)$
$(\circledR \vdash)$	$\mathcal{A} \vdash \mathcal{B} \;\}\; x\circledR\mathcal{A} \vdash x\circledR\mathcal{B}$		
$(\circledR \oslash)$	$\eta\circledR\mathcal{A} \vdash \mathcal{B} \;\{\}\; \mathcal{A} \vdash \mathcal{B}\oslash\eta$	$(\circledR\ n[])$	$x\circledR y[\mathcal{A}] \dashv\vdash y[x\circledR\mathcal{A}]$ $(x \neq y)$
$(\oslash \neg)$	$(\neg\mathcal{A})\oslash x \dashv\vdash \neg(\mathcal{A}\oslash x)$	$(\oslash\ n[])$	$y[\mathcal{A}]\oslash x \vdash y[\mathcal{A}\oslash x]$ $(x \neq y)$
$(\oslash \triangleright\mathbf{F})$	$\mathcal{A}^{\mathbf{F}}\oslash x \dashv\vdash \mathcal{A}^{\mathbf{F}}$	$(\oslash\ n[])$	$x[\mathcal{A}]\oslash x \vdash \mathbf{F}$
$(\circledR\ \mathbf{0})$	$x\circledR\mathbf{0} \dashv\vdash \mathbf{0}$		
$(\oslash\ \mathbf{0})$	$\mathbf{0}\oslash x \vdash \mathbf{0}$ $\square$		

From the rules that we have validated in Proposition 3-1, we can derive a large collection of facts by logical deduction, including the following:

3-2 Logical Corollaries (Case Analysis)

Let $C\!/$ be a classical predicate (typically, $C\!/$ is a side condition of the form $x \neq y$).

$(\mathbf{CA} \mid \wedge)$	$(C\!/ \wedge \mathcal{A}) \mid (C\!/ \wedge \mathcal{B}) \dashv\vdash C\!/ \wedge (\mathcal{A} \mid \mathcal{B})$	
$(\mathbf{CA} \mid \Rightarrow)$	$(C\!/ \Rightarrow \mathcal{A}) \mid (C\!/ \Rightarrow \mathcal{B}) \vdash C\!/ \Rightarrow (\mathcal{A} \mid \mathcal{B})$	
$(\mathbf{CA}\ n[]\ \wedge)$	$z[C\!/ \wedge \mathcal{A}] \dashv\vdash C\!/ \wedge z[\mathcal{A}]$	where z may occur in $C\!/$
$(\mathbf{CA}\ \circledR\ \wedge)$	$z\circledR(C\!/ \wedge \mathcal{A}) \dashv\vdash C\!/ \wedge z\circledR\mathcal{A}$	where z may occur in $C\!/$ $\square$

3-3 Logical Corollaries (Revelation)

$(\oslash)$	$\mathcal{A}\oslash x\oslash x \dashv\vdash \mathcal{A}\oslash x$	$(\oslash \mid)$	$\mathcal{A}\oslash x \mid x\circledR(\mathcal{B}\oslash x) \vdash (\mathcal{A} \mid \mathcal{B})\oslash x$
$(\oslash\ \oslash)$	$\mathcal{A}\oslash y\oslash x \vdash \mathcal{A}\oslash x\oslash y$		$(\mathcal{A} \mid \mathcal{B})\oslash x \vdash \mathcal{A}\oslash x \mid \mathcal{B}\oslash x$
$(\circledR \oslash \mathrm{R})$	$\mathcal{A} \vdash (x\circledR\mathcal{A})\oslash x$	$(\oslash\ \circledR)$	$\mathcal{A}\oslash x \vdash (x\circledR\mathcal{A})\oslash x$
	$x\circledR\mathcal{A} \vdash (x\circledR\mathcal{A})\oslash x$	hence:	$x\circledR(\mathcal{A}\oslash x) \vdash x\circledR\mathcal{A}$
$(\circledR \oslash \mathrm{L})$	$x\circledR(\mathcal{A}\oslash x) \vdash \mathcal{A}$	$(\circledR \wedge \oslash)$	$x\circledR(\mathcal{A} \wedge \mathcal{B}\oslash x) \dashv\vdash x\circledR\mathcal{A} \wedge \mathcal{B}$
	$x\circledR(\mathcal{A}\oslash x) \vdash \mathcal{A}\oslash x$	hence:	$x\circledR(\mathcal{B}\oslash x) \dashv\vdash x\circledR\mathbf{T} \wedge \mathcal{B}$
$(\circledR \oslash \mid)$	$x\circledR((\mathcal{A} \mid \mathcal{B})\oslash x)$	$(\circledR \vee \oslash)$	$x\circledR(\mathcal{A} \vee \mathcal{B}\oslash x) \vdash x\circledR\mathcal{A} \vee \mathcal{B}$
	$\dashv\vdash x\circledR(\mathcal{A}\oslash x) \mid x\circledR(\mathcal{B}\oslash x)$	$(\oslash \vdash)$	$\mathcal{A} \vdash \mathcal{B} \;\}\; \mathcal{A}\oslash x \vdash \mathcal{B}\oslash x$
$(\circledR \mid \circledR)$	$x\circledR(x\circledR\mathcal{A} \mid x\circledR\mathcal{B}) \dashv\vdash x\circledR\mathcal{A} \mid x\circledR\mathcal{B}$	$(\circledR \oslash \mid)$	$x\circledR((\mathcal{A} \mid \mathcal{B})\oslash x) \dashv\vdash x\circledR(\mathcal{A}\oslash x) \mid x\circledR(\mathcal{B}\oslash x)$
	$x\circledR\mathcal{A} \mid x\circledR\mathcal{B} \dashv\vdash (x\circledR\mathcal{A} \mid x\circledR\mathcal{B})\oslash x$	$(\circledR \oslash \mid)$	$x\circledR((\mathcal{A} \mid \mathcal{B})\oslash x) \vdash (x\circledR\mathcal{A})\oslash x \mid (x\circledR\mathcal{B})\oslash x$
$(\mid \circledR \oslash)$	$x\circledR\mathcal{A} \mid x\circledR(\mathcal{B}\oslash x) \vdash x\circledR(\mathcal{A} \mid \mathcal{B})$	$(\circledR \wedge \mid)$	$x\circledR\mathbf{T} \wedge (\mathcal{A} \mid \mathcal{B}) \dashv\vdash (x\circledR\mathbf{T} \wedge \mathcal{A}) \mid (x\circledR\mathbf{T} \wedge \mathcal{B})$
$(\oslash \mid \circledR)$	$\mathcal{A}\oslash x \mid x\circledR\mathcal{B} \vdash (\mathcal{A} \mid x\circledR\mathcal{B})\oslash x$	$(\circledR \Rightarrow \mid)$	$(x\circledR\mathbf{T}\Rightarrow\mathcal{A}) \mid (x\circledR\mathbf{T}\Rightarrow\mathcal{B}) \vdash x\circledR\mathbf{T} \Rightarrow (\mathcal{A} \mid \mathcal{B})$
$(\circledR \vee)$	$x\circledR(\mathcal{A} \vee \mathcal{B}) \dashv\vdash x\circledR\mathcal{A} \vee x\circledR\mathcal{B}$	$(\oslash\ n[])$	$y[\mathcal{A}]\oslash x \dashv\vdash y[\mathcal{A}\oslash x]$ $(x \neq y)$
$(\circledR \wedge)$	$x\circledR(\mathcal{A} \wedge \mathcal{B}) \vdash x\circledR\mathcal{A} \wedge x\circledR\mathcal{B}$	$(\oslash\ n[])$	$x[\mathcal{A}]\oslash x \dashv\vdash \mathbf{F}$
hence:	$x\circledR\mathcal{A} \dashv\vdash x\circledR\mathcal{A} \wedge x\circledR\mathbf{T}$	$(@\ \circledR)$	$(x\circledR\mathcal{A})@x \dashv\vdash \mathbf{F}$
$(\circledR\ \mathbf{F})$	$x\circledR\mathbf{F} \vdash \mathbf{F}$	$(@\ \circledR \neq)$	$(x\circledR\mathcal{A})@y \dashv\vdash x\circledR(\mathcal{A}@y)$ $(x \neq y)$

$(\oslash\ \mathbf{T})$ $\quad \mathbf{T} \dashv\vdash \mathbf{T}\oslash x$

$(\oslash\ \mathbf{F})$ $\quad \mathbf{F}\oslash x \dashv\vdash \mathbf{F}$

$(\oslash\ \vee)$ $\quad (\mathcal{A} \vee \mathcal{B})\oslash x \dashv\vdash \mathcal{A}\oslash x \vee \mathcal{B}\oslash x$

$(\oslash\ \wedge)$ $\quad (\mathcal{A} \wedge \mathcal{B})\oslash x \dashv\vdash \mathcal{A}\oslash x \wedge \mathcal{B}\oslash x$

$(\oslash\ \mathbf{0})$ $\quad \mathbf{0} \vdash \mathbf{0}\oslash x$

$(\circledR\ \neg\mathbf{0})$ $\quad x\circledR\neg\mathbf{0} \vdash \neg\mathbf{0}$

$(\circledR\ \oslash\ n[])$ $\quad x\circledR(y[\mathcal{A}]\oslash x) \dashv\vdash y[x\circledR(\mathcal{A}\oslash x)] \quad (x \neq y)$

$(\circledR\ \wedge\ n[])$ $\quad x\circledR\mathbf{T} \wedge y[\mathcal{A}] \dashv\vdash y[x\circledR\mathbf{T} \wedge \mathcal{A}] \quad (x \neq y)$

$(\oslash\ \circledR\ \neq)$ $\quad x\circledR(\mathcal{A}\oslash y) \vdash (x\circledR\mathcal{A})\oslash y$

$(\circledR\ \exists)$ $\quad \exists x.y\circledR\mathcal{A} \dashv\vdash y\circledR\exists x.\mathcal{A}$ $\quad$ where $x \neq y$

$(\circledR\ \forall)$ $\quad y\circledR\forall x.\mathcal{A} \vdash \forall x.y\circledR\mathcal{A}$ $\quad$ where $x \neq y$ $\quad \square$

Remark. The derived rule $(\circledR\ \neg\mathbf{0})$ says that if we reveal a restricted name and find non-**0**, then the original process is also non-**0**. That is, non-**0**-ness cannot be hidden by restriction. Consider, for example, the process $P = (\nu n)n[]$. Under many standard behavioral equivalences $\approx$ we have $P \approx \mathbf{0}$ [11]. However, we have $P \models n\circledR\neg\mathbf{0}$, and hence by $(\circledR\ \neg\mathbf{0})$, we have that $P \models \neg\mathbf{0}$. This example shows quite clearly that our logic is finer than standard behavioral equivalences, and that it can inspect the structure of restricted processes. $\square$

4 Fresh-Name Quantifier

In this section we define a formula, $!x.\mathcal{A}$, with the meaning "for fresh x, $\mathcal{A}$ holds". Here, "fresh" means, informally, distinct from any name that might clash with an existing name.

The set of free (i.e., non-fresh) names that occur in a process or formula is always finite; hence sets of fresh names are always cofinite. (A cofinite set is the complement of a finite set with respect to an infinite universe, which, in our case, is the countable universe of names Λ.) If there is a suitable fresh x, then there are infinitely many of them, since a fresh name can be replaced by any other fresh name. Therefore, "freshness" can be expressed formally as the existence of a cofinite set of interchangeable names [10]. We use $Fin(S)$ for the collection of finite subsets of a set S.

4.1 The Gabbay-Pitts Property

We would like to obtain the following property for $!x.\mathcal{A}$:

$$P \models !x.\mathcal{A} \quad \Leftrightarrow \quad \exists m{\in}\Lambda.\ m{\notin}fn(P,\mathcal{A}) \wedge P \models \mathcal{A}\{x{\leftarrow}m\}$$

That is, $P \models !x.\mathcal{A}$ iff there exists a fresh name m such that $P \models \mathcal{A}\{x{\leftarrow}m\}$.

This definition is given by existential quantification over fresh names. Remarkably, there is an equivalent definition based on universal quantification. The equivalence of these two definitions is based on a deep property of the logic (Lemma 2-3), and will be used to great effect later. We state the equivalence as follows: there exists a fresh name m such that $P \models \mathcal{A}\{x{\leftarrow}m\}$, if and only if for all fresh names m we have $P \models \mathcal{A}\{x{\leftarrow}m\}$:

4-1 Proposition (Gabbay-Pitts Property)

$\forall P{\in}\Pi, \mathcal{A}{\in}\Phi, N{\in}Fin(\Lambda).$

$N \supseteq fn(P,\mathcal{A}) \wedge fv(\mathcal{A}) \subseteq \{x\} \Rightarrow$

$(\exists m{\in}\Lambda.\ m{\notin}N \wedge P \models \mathcal{A}\{x{\leftarrow}m\}) \quad \Leftrightarrow \quad (\forall m{\in}\Lambda.\ m{\notin}N \Rightarrow P \models \mathcal{A}\{x{\leftarrow}m\})$

Proof

Assume $N \supseteq fn(P,\mathcal{A})$ and $fv(\mathcal{A}) \subseteq \{x\}$.

Case $\Leftarrow$) Assume $\forall m{\in}\Lambda.\ m{\notin}N \Rightarrow P \models \mathcal{A}\{x{\leftarrow}m\}$. Since N is finite and Λ is infinite, there is a $p{\in}\Lambda$ such that $p{\notin}N$. Then, by assumption, $P \models \mathcal{A}\{x{\leftarrow}p\}$. We have shown $(\exists p{\in}\Lambda.\ p{\notin}N \wedge P \models \mathcal{A}\{x{\leftarrow}p\})$.

Case $\Rightarrow$) Assume $\exists m{\in}\Lambda.\ m{\notin}N \wedge P \models \mathcal{A}\{x{\leftarrow}m\}$; in particular, $m{\notin}fn(P,\mathcal{A})$. Take any $p{\in}\Lambda$ and assume $p{\notin}N$. If $p{=}m$ we have by assumption that $P \models \mathcal{A}\{x{\leftarrow}\ p\}$. Otherwise, if $p{\neq}m$ then $p{\notin}N{\cup}\{m\}$; since $fn(P,\mathcal{A}\{x{\leftarrow}m\}) \subseteq N{\cup}\{m\}$, we have that $p{\notin}fn(P,\mathcal{A}\{x{\leftarrow}m\})$. By applying Lemma 2-3 to the assumption $P \models \mathcal{A}\{x{\leftarrow}m\}$ we obtain $P\{m{\leftarrow}p\} \models \mathcal{A}\{x{\leftarrow}m\}\{m{\leftarrow}p\}$; that is, $P \models \mathcal{A}\{x{\leftarrow}p\}$. In both cases, we have shown that $(\forall p{\in}\Lambda.\ p{\notin}N \Rightarrow P \models \mathcal{A}\{x{\leftarrow}p\})$. $\square$

4.2 A Gabbay-Pitts Logical Rule

We now want to formulate a Gabbay-Pitts property similar to Proposition 4-1, but expressible within the logic. We are going to use extensively the idiom $x\#N \wedge x\circledR\mathbf{T}$, for a quantified variable x. The first part of this conjunction says that the name x is fresh with respect to a given set of names N that usually includes the set of free names of a formula of interest. The second part says that x is fresh in the "underlying process", because $P \vDash n\circledR\mathbf{T}$ iff $n \notin fn(P)$. For a suitable choice of N, the whole conjunction can be understood as saying that x is "completely fresh", both at the formula and process level, in a given situation.

Notation

- For $N \in Fin(\Lambda \cup \vartheta)$ we define the formula $\eta\#N \triangleq \bigwedge_{\mu \in N}(\eta \neq \mu)$.
  For any P and closed $m\#N$, we have $P \vDash m\#N$ iff $m \notin N$.
- Let $fnv(\mathcal{A}) \triangleq fn(\mathcal{A}) \cup fv(\mathcal{A})$, so that $fnv(\mathcal{A}) \in Fin(\Lambda \cup \vartheta)$

With this understanding, the following proposition states the single rule (schema) that we add to our logic in order to capture "freshness", and establishes its soundness. Note that this rule holds for open formulas.

4-2 Proposition (Validity: Gabbay-Pitts)
(GP) $\vdash \exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A} \dashv\vdash \forall x.\, (x\#N \wedge x\circledR\mathbf{T}) \Rightarrow \mathcal{A}$
where $N \in Fin(\Lambda \cup \vartheta)$ and $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N$

Proof

Assume $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N$. We need to show that the sequent is valid, that is that $\forall \varphi \in (fv(\mathcal{A})\text{-}\{x\}) \to \Lambda,\ P \in \Pi.\ P \vDash (\exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A})_\varphi \Leftrightarrow P \vDash (\forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A})_\varphi$.

(1) $\vdash \exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A} \vdash \forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A}$
Take any $\varphi \in (fv(\mathcal{A})\text{-}\{x\}) \to \Lambda$ and $P \in \Pi$, and assume $P \vDash (\exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A})_\varphi$. That is, assume $\exists m \in \Lambda.\ m \notin N_\varphi \cup fn(P) \wedge P \vDash \mathcal{A}_\varphi\{x \leftarrow m\}$, where $N_\varphi \cup fn(P) \supseteq fn(P, \mathcal{A}_\varphi)$ and $fv(\mathcal{A}_\varphi) \subseteq \{x\}$. By Proposition 4-1, we obtain $\forall m \in \Lambda.\ m \notin N_\varphi \cup fn(P) \Rightarrow P \vDash \mathcal{A}_\varphi\{x \leftarrow m\}$, that is $P \vDash (\forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A})_\varphi$.

(2) $\vdash \forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A} \vdash \exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A}$
Take any $\varphi \in (fv(\mathcal{A})\text{-}\{x\}) \to \Lambda$ and $P \in \Pi$ and assume $P \vDash (\forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A})_\varphi$; that is assume $(\forall m \in \Lambda.\ m \notin N_\varphi \cup fn(P) \Rightarrow P \vDash \mathcal{A}_\varphi\{x \leftarrow m\})$, where $N_\varphi \cup fn(P) \supseteq fn(P, \mathcal{A}_\varphi)$ and $fv(\mathcal{A}_\varphi) \subseteq \{x\}$. By Proposition 4-1, we obtain $\exists m \in \Lambda.\ m \notin N_\varphi \cup fn(P) \wedge P \vDash \mathcal{A}_\varphi\{x \leftarrow m\}$, that is $P \vDash (\exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A})_\varphi$. □

Remark. (GP) gives us a way to prove that $\forall x.\mathcal{A} \vdash \exists x.\mathcal{A}$. This depends on the fact that the set of names is non-empty, and is obviously not derivable from the normal quantifier rules. Take $N = fnv(\mathcal{A})\text{-}\{x\}$. Starting from $\mathcal{A} \vdash \mathcal{A}$, by right weakening and quantifier introduction we obtain $\forall x.\, \mathcal{A} \vdash \forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A}$. Again starting from $\mathcal{A} \vdash \mathcal{A}$, by left weakening and quantifier introduction we obtain $\exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A} \vdash \exists x.\, \mathcal{A}$. By (GP) we have $\forall x.\, x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A} \vdash \exists x.\, x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A}$. Hence, by transitivity we obtain $\forall x.\, \mathcal{A} \vdash \exists x.\, \mathcal{A}$. □

4.3 Fresh-Name Quantifier

Without extending the syntax of our logic, we can define quantification over fresh names, $!x.\mathcal{A}$, as follows:

4-3 Definition (Fresh-Name Quantifier)
$!x.\mathcal{A} \triangleq \exists x.\, x\#(fnv(\mathcal{A})\text{-}\{x\}) \wedge x\circledR\mathbf{T} \wedge \mathcal{A}$ □

Hence $fn(!x.\mathcal{A}) = fn(\mathcal{A})$ and $fv(!x.\mathcal{A}) = fv(\mathcal{A})\text{-}\{x\}$.

Note that the right-hand side of this definition depends on the set of free names and variables of $\mathcal{A}$. Therefore, this is not a definition within the logic, but rather a meta-theoretical definition

(or abbreviation) that should always be understood in its expanded form. Any general theorem or derived rule involving $!x.\mathcal{A}$ will in fact be a schematic theorem or rule with respect to the free names and variables of $\mathcal{A}$, in the same way that (GP) is a rule schema.

By (GP) (Proposition 4-2) we have:

$$!x.\mathcal{A} \dashv\vdash \forall x.\ x\#fnv(\mathcal{A})\text{-}\{x\} \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A}$$

In terms of satisfaction, we obtain:

4-4 Lemma ($P \models !x.\mathcal{A}$)

$P \models !x.\mathcal{A}$
iff $\exists m\in\Lambda.\ m\notin fn(P,\mathcal{A}) \wedge P \models \mathcal{A}\{x\leftarrow m\}$
iff $\forall m\in\Lambda.\ m\notin fn(P,\mathcal{A}) \Rightarrow P \models \mathcal{A}\{x\leftarrow m\}$ □

Therefore, $!x.\mathcal{A}$ can be understood as saying either that there is a fresh name x such that $\mathcal{A}$ holds, or that for any fresh name x we have that $\mathcal{A}$ holds. These formulations are equivalent because of the cofinite nature of sets of fresh names. If there is a suitably fresh x such that $\mathcal{A}$ holds, then any other fresh name will work equally well, so all fresh names will work. Conversely, if for all suitably fresh names $\mathcal{A}$ holds, since any set of fresh names is (cofinite and hence) non-empty, there exists a fresh name for which $\mathcal{A}$ holds.

Remark. The meaning of $!x.\mathcal{A}$ when $\mathcal{A}$ has free variables other than x is subtle. When we write $!x.\ ...n...$ we intend x to be fresh w.r.t. any existing name, and in particular n; similarly, when we write $!x.\ ...y...$ we intend x to be fresh with respect to any name denoted by y. Consider $!y.\ !x.\ y=x$; this formula should not be valid. In fact, it is contradictory because, by definition, it means, $\exists y.\ y\circledR\mathbf{T} \wedge \exists x.\ x\neq y \wedge x\circledR\mathbf{T} \wedge y=x$. Similarly, $\forall y.\ !x.\ y=x$ and $\exists y.\ !x.\ y=x$ are contradictory. (Instead, $!x.\ \exists y.\ x=y$ is valid.) □

The following rules are now derivable entirely within the logic:

4-5 Logical Corollaries (Fresh-Name Quantifier)

(! ∃)	$\vdash !x.\mathcal{A} \dashv\vdash \exists x.\ x\#N \wedge x\circledR\mathbf{T} \wedge \mathcal{A}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x\notin N$
(! ∀)	$\vdash \forall x.\ x\#N \wedge x\circledR\mathbf{T} \Rightarrow \mathcal{A} \dashv\vdash !x.\mathcal{A}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x\notin N$
(! ¬)	$\vdash \neg !x.\mathcal{A} \dashv\vdash !x.\neg\mathcal{A}$	
(! |)	$\vdash !x.(\mathcal{A} \mid \mathcal{B}) \dashv\vdash (!x.\mathcal{A}) \mid (!x.\mathcal{B})$	
(! ⊢)	$\mathcal{A} \vdash \mathcal{B} \vdash !x.\mathcal{A} \vdash !x.\mathcal{B}$	
(! *fv*)	$\vdash !x.\mathcal{A} \dashv\vdash \mathcal{A}$	where $x\notin fv(\mathcal{A})$
(! *n*[])	$\vdash !x.y[\mathcal{A}] \dashv\vdash y[!x.\mathcal{A}]$	where $x\neq y$
(! R)	$\mathcal{A} \wedge x\#N \wedge x\circledR\mathbf{T} \vdash \mathcal{B} \vdash \mathcal{A} \vdash !x.\mathcal{B}$	where $N \supseteq fnv(\mathcal{B})\text{-}\{x\}$ and $x \notin N \cup fv(\mathcal{A})$
(! L)	$\mathcal{A} \wedge x\#N \wedge x\circledR\mathbf{T} \vdash \mathcal{B} \vdash !x.\mathcal{A} \vdash \mathcal{B}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N \cup fv(\mathcal{B})$
(! E)	$\mathcal{A} \vdash !x.\mathcal{B};\ \mathcal{B} \wedge x\#N \wedge x\circledR\mathbf{T} \vdash C \vdash \mathcal{A} \vdash C$	where $N \supseteq fnv(\mathcal{B})\text{-}\{x\}$ and $x\notin N \cup fv(C)$ □

Remark. Of particular interest (and difficulty) is the distribution of ! over |, rule (! |):

$$\vdash !x.(\mathcal{A} \mid \mathcal{B}) \dashv\vdash (!x.\mathcal{A}) \mid (!x.\mathcal{B})$$

Distribution over | holds in one direction for universal quantification, in the other direction for existential quantification, and in both directions for fresh-name quantification. This rule can be understood informally as follows (this is a sketch of the formal derivation). In the left-to-right direction we use the existential interpretation of !. Take any P; if $P \models !x.(\mathcal{A} \mid \mathcal{B})$ then there are a fresh name x and processes P',P'' such that $P \equiv P' \mid P''$ and $P' \models \mathcal{A}$ and $P'' \models \mathcal{B}$. Hence, there is a fresh name x such that $P' \models \mathcal{A}$ and again a fresh name x such that $P'' \models \mathcal{B}$; that is, $P' \models !x.\mathcal{A}$ and $P'' \models !x.\mathcal{B}$. Therefore, $P \equiv P' \mid P'' \models (!x.\mathcal{A}) \mid (!x.\mathcal{B})$. In the right-to-left direction we use the universal interpretation of !. Take any P; if $P \models (!x.\mathcal{A}) \mid (!x.\mathcal{B})$ then there are processes P',P'' such that $P \equiv P' \mid P''$ and $P' \models !x.\mathcal{A}$ and $P'' \models !x.\mathcal{B}$. This means that for all names x' fresh in P' and $\mathcal{A}$, we have $P' \models \mathcal{A}\{x\leftarrow x'\}$ and for all names x'' fresh in P'' and $\mathcal{B}$, we have $P'' \models \mathcal{B}\{x\leftarrow x''\}$.

Now, for all names y that are fresh in P', $\mathcal{A}$, P'', $\mathcal{B}$; we have that $P' \vDash \mathcal{A}\{x \leftarrow y\}$ and $P'' \vDash \mathcal{B}\{x \leftarrow y\}$. That is, $P \equiv P' \mid P'' \vDash\ !\,y.(\mathcal{A}\{x \leftarrow y\} \mid \mathcal{B}\{x \leftarrow y\}) = !\,x.(\mathcal{A} \mid \mathcal{B})$. □

5 Hidden-Name Quantifier

As discussed in the introduction, a *hidden-name quantifier* should be a construct of the logic that allows us to talk about restricted names in processes. We would like to define a formula $(\nu x)\mathcal{A}$ to mean, informally, that "for hidden name x" (hidden in the underlying process), $\mathcal{A}$ holds. The intention is that there should be some correspondence between the binder (νx) in the formula, and a binder (νn) in a process that satisfies the formula. We take:

5-1 Definition (Hidden-Name Quantifier)
$(\nu x)\mathcal{A} \triangleq\ !\,x.x \circledR \mathcal{A}$ □

Hence $fn((\nu x)\mathcal{A}) = fn(\mathcal{A})$ and $fv((\nu x)\mathcal{A}) = fv(\mathcal{A})\text{-}\{x\}$. Moreover, by definition of $!$:

$$(\nu x)\mathcal{A} \;=\; \exists x.\ x \# fnv(\mathcal{A})\text{-}\{x\} \wedge x \circledR \mathbf{T} \wedge x \circledR \mathcal{A}$$

and, because of Logical Corollary 3-3($\circledR \wedge$), we can simplify this to:

$$(\nu x)\mathcal{A} \;\dashv\vdash\; \exists x.\ x \# fnv(\mathcal{A})\text{-}\{x\} \wedge x \circledR \mathcal{A}$$

In terms of satisfaction, we obtain:

5-2 Lemma ($P \vDash (\nu x)\mathcal{A}$)
$P \vDash (\nu x)\mathcal{A}$ iff
$\exists m \in \Lambda.\ m \notin fn(P,\mathcal{A}) \wedge \exists P' \in \Pi.\ P \equiv (\nu m)P' \wedge P' \vDash \mathcal{A}\{x \leftarrow m\}$ □

We have in fact experimented with several plausible definitions for the hidden-name quantifier, before converging on the one above. We have found that the following property, (νx-proper), distinguishes the definition above from other definitions of $(\nu x)\mathcal{A}$ that turned out to be unsatisfactory or flawed:

5-3 Proposition (νx-proper)
For all $n \in \Lambda$, $x \in \vartheta$, $P \in \Pi$, and closed $\mathcal{A} \in \Phi$:
$n \notin fn(P) \wedge P \vDash (\nu x)(\mathcal{A}\{n \leftarrow x\}) \quad \Leftrightarrow \quad \exists P' \in \Pi.\ P \equiv (\nu n)P' \wedge P' \vDash \mathcal{A}$ □

Corollary: $P' \vDash \mathcal{A} \;\Rightarrow\; (\nu n)P' \vDash (\nu x)(\mathcal{A}\{n \leftarrow x\})$. □

This property can be written in logical form as $n \circledR \mathbf{T} \wedge (\nu x)(\mathcal{A}\{n \leftarrow x\}) \dashv\vdash n \circledR \mathcal{A}$, for all n.

Remark. It is natural to first consider the simpler property:

$$(\nu n)P' \vDash (\nu x)(\mathcal{A}\{n \leftarrow x\}) \quad \Leftrightarrow \quad P' \vDash \mathcal{A} \qquad (\nu x\text{-}1)$$

The $\Leftarrow$ direction is equivalent to (νx-proper$\Leftarrow$). However, the $\Rightarrow$ direction is inconsistent with the fundamental Lemma 2-1. Start with $n[] \vDash n[]$. By (νx-1$\Leftarrow$) we obtain $(\nu n)n[] \vDash (\nu x)x[]$. Since $(\nu n)n[] \equiv (\nu n)(\nu n)n[]$, by Lemma 2-1 we obtain that $(\nu n)(\nu n)n[] \vDash (\nu x)x[]$. Then, by ($\nu x-1\Rightarrow$) we obtain $(\nu n)n[] \vDash n[]$, that is $(\nu n)n[] \equiv n[]$, which is contradictory by Lemma 2-2(1). The problem is that we cannot expect a (νx) in the formula to match any (νn) in the process, but only an appropriate one. Hence the refined statement of (νx-proper). □

The following rules for $(\nu x)\mathcal{A}$ can be derived by the rules for revelation and fresh-name.

5-4 Logical Corollaries (Hidden-Name Quantifier)

($\nu\ \forall$)	$\{\ \forall x.\ (x \# N \wedge x \circledR \mathbf{T}) \Rightarrow x \circledR \mathcal{A} \dashv\vdash (\nu x)\mathcal{A}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N$
($\nu\ \exists$)	$\{\ (\nu x)\mathcal{A} \dashv\vdash \exists x.\ x \# N \wedge x \circledR \mathbf{T} \wedge x \circledR \mathcal{A}$ $\dashv\vdash \exists x.\ x \# N \wedge x \circledR \mathcal{A}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N$
(ν R)	$\mathcal{A} \wedge x \# N \wedge x \circledR \mathbf{T} \vdash x \circledR \mathcal{B}\ \}\ \mathcal{A} \vdash (\nu x)\mathcal{B}$	where $N \supseteq fnv(\mathcal{B})\text{-}\{x\}$ and $x \notin N \cup fv(\mathcal{A})$
(ν L)	$x \circledR \mathcal{A} \wedge x \# N \vdash \mathcal{B}\ \}\ (\nu x)\mathcal{A} \vdash \mathcal{B}$	where $N \supseteq fnv(\mathcal{A})\text{-}\{x\}$ and $x \notin N \cup fv(\mathcal{B})$

(ν E) $\mathcal{A} \vdash (\nu x)\mathcal{B};\ x \circledR \mathcal{B} \wedge x\#N \vdash C \ \}\ \mathcal{A} \vdash C$ where $N \supseteq fnv(\mathcal{B})\text{-}\{x\}$ and $x \notin N \cup fv(C)$

(ν ⊢) $\mathcal{A} \vdash \mathcal{B}\ \}\ (\nu x)\mathcal{A} \vdash (\nu x)\mathcal{B}$

(ν *fv*) $\}\ (\nu x)(\mathcal{A} \oslash x) \dashv\vdash \mathcal{A}$ where $x \notin fv(\mathcal{A})$

(ν *fv*) $\}\ \mathcal{A} \vdash (\nu x)\mathcal{A}$ where $x \notin fv(\mathcal{A})$

(ν ®) $\}\ (\nu x)(x \circledR \mathcal{A}) \dashv\vdash (\nu x)\mathcal{A}$

(ν ⊘) $\}\ (\nu x)(\mathcal{A} \oslash x) \vdash (\nu x)\mathcal{A}$

(ν **0**) $\}\ (\nu x)\mathbf{0} \dashv\vdash \mathbf{0}$

(ν *n*[]) $\}\ (\nu x)y[\mathcal{A}] \dashv\vdash y[(\nu x)\mathcal{A}]$ where $x \neq y$

(ν |) $\}\ (\nu x)(\mathcal{A} \mid x \circledR \mathcal{B}) \dashv\vdash ((\nu x)\mathcal{A}) \mid ((\nu x)\mathcal{B})$

(ν ⊘ |) $\}\ (\nu x)(\mathcal{A} \oslash x) \mid (\nu x)(\mathcal{B} \oslash x) \dashv\vdash (\nu x)((\mathcal{A} \mid \mathcal{B}) \oslash x)$ □

Remark. We obtain $\forall x.\ x \circledR \mathcal{A} \vdash (\nu x)\mathcal{A} \vdash \exists x.\ x \circledR \mathcal{A}$. However, there are no interesting rules for $\neg(\nu x)\mathcal{A}$. □

Remark. This fails:

$\}\ (\nu x)\mathcal{A} \vdash \mathcal{A}$ where $x \notin fv(\mathcal{A})$

because $(\nu n)(n[] \mid n[]) \vDash\ !x.x \circledR (\neg\mathbf{0} \mid \neg\mathbf{0})$ but $(\nu n)(n[] \mid n[]) \nvDash \neg\mathbf{0} \mid \neg\mathbf{0}$. This is ®'s fault, not !'s: $n \circledR \mathcal{A} \vdash \mathcal{A}$ fails with the same counterexample. □

Example

As an example of a specification containing a hidden-name quantifier, consider a situation where a secret is shared by two locations *n* and *m*, but is not known outside those locations.

We can state this as follows (recall that $\copyright\eta \triangleq \neg\eta \circledR \mathbf{T}$ and that $P \vDash \copyright n$ iff $n \in fn(P)$):

$$(\nu x)\ (n[\copyright x] \mid m[\copyright x])$$

It reads: for a fresh *x*, the name *x* is known at *n* and *m*, and is restricted anywhere else.

Expanding the definitions, we obtain:

$P \vDash (\nu x)\ (n[\copyright x] \mid m[\copyright x])$

$\Leftrightarrow \exists r \in \Lambda.\ r \notin fn(P) \cup \{n,m\} \wedge \exists R', R'' \in \Pi.\ P \equiv (\nu r)(n[R'] \mid m[R'']) \wedge r \in fn(R') \wedge r \in fn(R'')$

The last line reads: *P* satisfies the specification iff there exists a name *r* that is fresh (not conflicting with *n* and *m* or public to *P*), such that *r* is known to the processes *R'* and *R"* located at *n* and *m*, and is restricted inside *P*.

Here is a simple example of an implementation of this specification:

$$P = (\nu p)\ (n[p[]] \mid m[p[]])$$

6 Related Work and Conclusions

We have introduced a logic for describing concurrent processes with restricted names. Most previous logics for concurrency have strived to describe properties that are invariant under some coarse process equivalence, such as bisimulation. Because of our original motivation in describing location structures in detail, the properties described by our logic are much finer, and are invariant only up to structural congruence (see also [14] for a recent characterization). Because of this, our logic is closely related to intuitionistic linear logic and to bunched logics: see [7] for a comparison. Our logic is unusual also because it handles variables ranging over a countable universe of names; these variables can be the subject of universal, existential, fresh-name, and hidden-name quantification.

Our logic is built directly out of a process model, so logical soundness is easy to check. Logical completeness is a much more difficult question. We do not expect the full logic to be complete with respect to our model (even for finite behaviors). Silvano Dal Zilio is investigating some small, complete fragments of the logic. So far, we have mostly tried to discover as many true logical facts as possible (a measure of which is, for example, to be able to embed other logics into

ours [7]), and to minimize the collection of basic rules. We have concentrated in particular on commutation and distribution properties of operators that can be useful in formal proofs.

In the present paper, fresh-name quantification is modeled after Gabbay and Pitts [10], adapted to our context; it provides logical rules for reasoning abstractly about freshness. Hidden-name quantification is obtained by combining fresh-name quantification with a revelation operator (not a quantifier) for revealing restricted process names. Most novel axioms have to do with revelation; they often reflect and resemble well-known properties of π-calculus restriction. Technically, we have added to our previous ambient logic just the revelation operator (and its adjunct) and an axiom schema expressing the Gabbay-Pitts property. In particular, fresh-name quantification, hidden-name quantification, and their properties, are derived.

Recently, we have become aware of related work by Luís Caires (both [3] and more recent unpublished work). Our aims are quite similar, but we are currently using different formal techniques; we are in the process of comparing results.

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