FRACTAL WAVELET CODING USING A RATE-DISTORTION CONSTRAINT

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ABSTRACT

We propose a hybrid fractal wavelet coder (FWC) which adopts adaptive fractal prediction in the wavelet domain and wavelet coding in this research. This new compression scheme uses wavelet coding as the core component, and applies contractive mapping (or the so-called fractal prediction) whenever it offers additional rate savings. A model based rate-distortion (R-D) criterion of low complexity is used to evaluate the fractal prediction efficiency and search for the best domain-range match. The superior R-D performance of FWC is demonstrated with extensive experimental results.

Keywords: image coding, rate distortion criterion, fractal contractive mapping, wavelet transform.

1. INTRODUCTION

The relationship between the fractal coder and the conventional transform-based coders has been studied by researchers recently. Generally speaking, contractive mapping can be viewed as an interscale prediction in the wavelet/ subband domain [1]. Despite all recent developments in fractal theory and fractal coder implementations, the rate-distortion (R-D) performance of fractal coders is still not competitive with the state-of-the-art transform-based coders such as the embedded zerotree wavelet coder (EZW) of Shapiro [5] and the layered zero wavelet coder of Taubman and Zakhor [6]. The inferior R-D performance becomes more transparent after the relationship between contractive mapping and the wavelet transform is better understood.

In this work, we continue to shed light on the relationship between the fractal contractive mapping and the wavelet coding, and provide insights into the design of a new image coder. Instead of using contractive mapping as a fundamental coding tool, we build a new coder on the foundation of the wavelet coder and apply contractive mapping adaptively as an auxiliary tool in regions where fractal prediction offers additional rate savings. The resulting hybrid fractal wavelet coder (FWC) gives a very superior R-D performance.

It is worthwhile to point out that a predictive pyramid coder (PPC) which predicts fine-scale wavelet coefficients by using those of the coarser scales was recently proposed by Rinaldo and Calvagno [4]. However, our scheme differs in two aspects. First, the interscale wavelet prediction in their coder is very different from the fractal prediction adopted in our coder. Second, PPC encodes a majority portion of the image by interscale wavelet prediction and a very small portion by wavelet coding. The situation is reversed with the proposed new coder. We use the wavelet coding as our primarily coding tool and adopt fractal prediction only when it is efficient.

The paper is organized as follows. Contractive mapping in the wavelet domain and its efficiency evaluation are studied in Section 2. Implementation details of FWC are given in Section 3. The performance of FWC is demonstrated with extensive experiments in Section 4. Some concluding remarks are given in Section 5.

2. FRACTAL PREDICTION AND EFFICIENCY EVALUATION

2.1. Contractive Mapping in the Wavelet Domain

The concept of fractal prediction among different wavelet subbands is derived from the fractal coding operation in the spatial domain. With a block-based spatial-domain fractal coder, image $\mathbf{f}$ is partitioned into a set of non-overlapping square range blocks $\{\mathbf{r_j}, j=1,\cdots,N_R\}$ of size $K_R \times K_R$:

$$\bigcup_{j=1}^{N_R} \mathbf{r_j} = \mathbf{f}, \quad \mathbf{r_i} \cap \mathbf{r_j} = \emptyset, \quad for \quad i \neq j \tag{1}$$

Each range block is matched with a domain block $\mathbf{d_k}$ of size $K_D \times K_D$. All domain blocks constitute a domain pool $\mathcal{D} = \{\mathbf{d_k}, k=1,\cdots,N_D\}$. Usually, the size of the domain block is twice as large as the range block, i.e. $K_D = 2\cdot K_R$. During the fractal coding process, domain block $\mathbf{d_k}$ is subsampled from size $K_D \times K_D$ down to size $K_R \times K_R$, isometrically transformed, and scaled and offset in the pixel magnitude. Mathematically, the process can be written as:

$$\hat{\mathbf{r_j}} = \tau_j(\mathbf{d_k}) = \alpha_j \Gamma_j(S_j(\mathbf{d_k})) + o_j, \tag{2}$$

where S_j, Γ_j, α_j and o_j denote spatial contraction, isometrical transformation, contrast scaling and luminance offset, respectively. Due to the use of spatial contraction S_j, fractal coding is also called *contractive mapping*.

Fractals exhibit self-similarity of signals across scales while wavelets are useful for multiscale signal analysis. Thus, it is interesting to investigate the fractal contractive mapping in the wavelet domain [1]. By transforming (2) into the Haar wavelet domain, the averaging and subsampling operator S_j is equivalent to move the domain block one-scale up. The isometric operator Γ_j is still a pixel shuffling operation

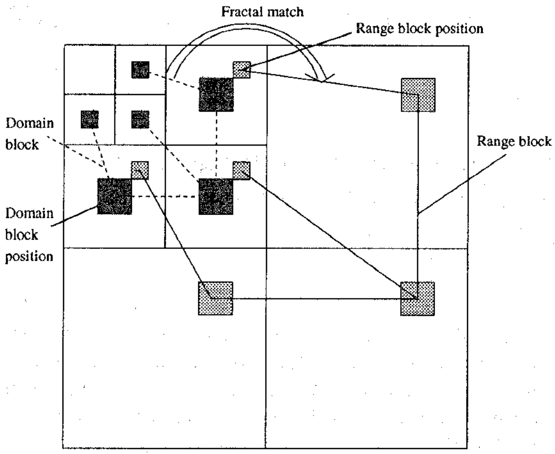

Figure 1: Contractive mapping in the wavelet domain

which now involves additional sign changes and inter subband coefficient switching [3]. The contrast scaling factor α_j is reduced by one half because of normalization, and the offset o_j relates only to the difference between the DC components of range and domain blocks. By ignoring the DC component, the contractive mapping in the wavelet domain actually establishes a prediction relation of the wavelet coefficients across scales, where the wavelet coefficients in the finer scales are shuffled and scaled to predict the wavelet coefficients in the coarser scales, as shown in Fig. 1. It is called *fractal prediction* in the wavelet domain. The mathematical formulation of fractal prediction can be written as:

$$\hat{\mathbf{r}}_{\mathbf{j}} = \tau_j(\mathbf{d}_{\mathbf{k}}) = \alpha_j \Gamma_j(\mathbf{d}_{\mathbf{k}}) \tag{3}$$

No more contractive operator S_j is needed explicitly, since it has been performed by the wavelet transform.

Although the above analysis is derived based on the Haar wavelet assumption, the result can be extended to other wavelet bases. The spatial matching relation (2) basically implies that the range block resembles the contractive mapping of a certain domain block in the spatial domain. The equivalent statement in the wavelet domain is that wavelet coefficients of finer scale subband can be predicted by the contractive mapping of those of the coarser scales. Even though the above result can be rigorously derived with the Haar basis, its validity also holds for other wavelet bases.

2.2. Fractal Efficiency Evaluation

Since fractal contractive mapping corresponds to an inter-scale prediction in the wavelet domain, it is natural to apply it to a conventional wavelet coder to construct a hybrid fractal wavelet coder. However, fractal prediction is not efficient for all image regions and should only be used adaptively. That is, we evaluate the required coding rate for each range block before and after fractal prediction, and adopt fractal prediction if there is a gain in coding rate saving.

In this research, a model-based rate-distortion (R-D) criterion is used for rate saving estimation. Suppose that range block $\mathbf{r_j}$ is constituted of N wavelet subblocks of sizes $S_1, S_2, \cdots, S_N$, respectively. Before fractal prediction, the variances of the wavelet subblocks are $\sigma^2_{b,1}, \sigma^2_{b,2}, \cdots, \sigma^2_{b,N}$. After fractal prediction, the variances of the wavelet subblocks are $\sigma^2_{a,1}, \sigma^2_{a,2}, \cdots, \sigma^2_{a,N}$. Besides, we need $R_o(j)$ bits to encode the wavelet contractive mapping, which is the overhead of fractal prediction. Thus, the expected coding rates before and after fractal prediction are:

$$R_b(j) = \frac{1}{2}\sum_{i=1}^{N}\log_2\frac{\kappa^2_{b,i}}{D} \quad R_a(j) = \frac{1}{2}\sum_{i=1}^{N}\log_2\frac{\kappa^2_{a,i}}{D} \tag{4}$$

$$\kappa^2_{b,i} = \begin{cases} \sigma^2_{b,i}, & \sigma^2_{b,i} > D \\ D, & \sigma^2_{b,i} \le D \end{cases} \quad \kappa^2_{a,i} = \begin{cases} \sigma^2_{a,i}, & \sigma^2_{a,i} > D \\ D, & \sigma^2_{a,i} \le D \end{cases} \tag{5}$$

where D is the coding quality control parameter which is roughly related to the coding PSNR via

$$D = 255^2 10^{-\frac{PSNR}{10}} \tag{6}$$

and κ^2_i is the *lower bounded variance* of σ^2_i. The quantity

$$R_s(j) = R_b(j) - R_a(j) - R_o(j), \tag{7}$$

is the expected coding rate reduction through the use of fractal prediction. Fractal prediction should be used only when $R_s(j)$ is positive. For more details of the model-based R-D criterion, we refer to [3].

3. FRACTAL WAVELET IMAGE CODER

Our proposed coder is a hybrid coder which adopts adaptive fractal prediction and wavelet coding. We use the wavelet coder as the primary coder and the fractal contractive mapping as an auxiliary tool which is applied only to regions with a positive rate saving. The coding process starts from the lowest resolution to the highest resolution. Detailed implementations are described below.

Step 1: Wavelet Decomposition of Images

We decompose the image by a 2-D pyramidal wavelet transform to depth $d = 5$ with the biorthogonal 9-7 tap spline filter. When performing the wavelet decomposition, the image is symmetrically reflected across boundaries.

Step 2: Fractal Range-Domain Search and Prediction Efficiency Evaluation

After wavelet decomposition, we search in the domain pool $\mathcal{D}$ for the optimal range-domain pair $\hat{\mathbf{r}}_{\mathbf{j}} = \tau_j(\mathbf{d}_{\mathbf{k}})$ with respect to each range block $\mathbf{r_j}$. For image coding, the *best* match is determined in the sense that it minimizes the overall coding rate. Thus, we minimize the rate saving (7) rather than the prediction mean square error (MSE) or the prediction mean absolute error (MAE). For range blocks with an expected coding rate before fractal prediction smaller than the description overhead (i.e. $R_b(j) < R_o(j)$), fractal prediction will not be used since the rate saving $R_s(j)$ is negative no matter what $R_a(j)$ is. This criterion is used as a screening process to avoid many unnecessary search operations.

After the search, we calculate the expected rate saving $R_s(j)$ and record the fractal prediction status $s(j)$ for each

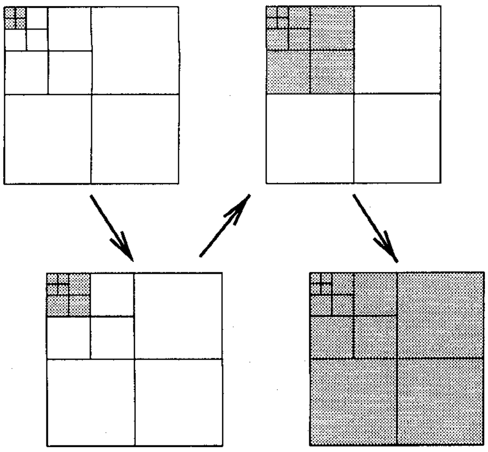

Figure 2: The proposed fractal wavelet coding process.

range block $\mathbf{r_j}$ as

$$s(j) = \begin{cases} 1, & R_s(j) > 0, \\ 0, & R_s(j) \leq 0 \end{cases} \tag{8}$$

If $R_s(j)$ is positive, fractal prediction for range block $\mathbf{r_j}$ is accepted, and prediction status $s(j)$ is set to 1. Otherwise, it is set to 0. All values of $s(j)$ are stored according to the position of the range block, leading to a small binary image which is called the prediction status image. This image is encoded using a 2-bit context adaptive arithmetic coder, where the context consists of prediction statuses of the left and upper blocks. For the range block using fractal prediction, the description of contractive mapping includes the position of the domain block $\mathbf{d_k}$, the type of the isometric operator Γ_j, and the quantized scaling factor α_j.

Step.3 Layered Zero Coding

After the fractal search, we use the layered zero coder (LZC) [6] to encode prediction residuals and wavelet coefficients associated with the non-fractal predicted range blocks in a top-down manner as shown in Fig. 2. We have made some minor modifications of LZC to adapt it to the fractal wavelet coding. The essence of LZC is that wavelet coefficients are quantized successively into layers, so that quantization outputs for every layer are simply binary values ('0's and '1's) which can be compressed very effectively by the context adaptive arithmetic coder. Note that LZC is selected to serve as the wavelet coder for its outstanding R-D performance and simplicity. However, it is by no means that the wavelet coder should be restricted to LZC, and other wavelet coders can also be used as a building module.

Our coding process starts from scale d. We first encode wavelet coefficients in scale d with LZC, and no fractal prediction for scale d is applied. Then, we move on to scale $d-1$, and use fractal prediction for wavelet coefficients from scales d to $d-1$. For the non-fractal predicted blocks, the prediction is set to 0 and the prediction residue is just the same as the original wavelet coefficients. We use again LZC to encode both the prediction residue and the non-fractal predicted wavelet coefficients in scale $d-1$. The process repeats until all prediction residues in scale 1 are encoded.

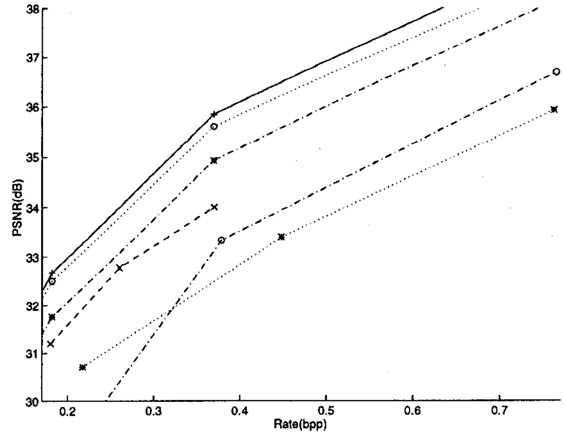

Figure 3: Rate-distortion coding results for Lena using the (a) FWC (the solid line with '+'), (b) LZC (the dotted line with 'o'), (c) EZW (the dash dotted line with '*'), (d) PPC (the dashed line with 'X'), (e) FRAC (the dotted line with '*'), (f) JPEG (the dash dotted line with 'o').

4. EXPERIMENTAL RESULTS

We compare the image coding results of the proposed fractal wavelet coder (FWC) with various other fractal and wavelet coders applied to Lena of size 512×512. The coders in comparison include the fractal coder (FRAC) [2], the predictive pyramid coder(PPC) [4], the embedded zerotree wavelet coder(EZW) [5], and the layered zero coder(LZC) [6]. Coding results of JPEG are also provided as a reference. Experimental results are shown in Table. 1 and the R-D performance curves are plotted in Fig. 3.

The performance of the spatial fractal coder (FRAC) is very poor even compared with the JPEG standard. FRAC outperforms JPEG at lower bit rates, but it cannot compete with JPEG at the middle to high bit rate range. Although the predictive pyramid coder (PPC), which is a fractal wavelet coder, significantly outperforms the JPEG standard, it is still not as competitive as the state-of-art wavelet coders such as EZW or LZC. Based on the experiment results of the Lena image, the proposed FWC gives the best performance (outperforming LZC by 0.1-0.4dB, EZW by 0.6-0.9dB, PPC by 1.5-2.0dB, JPEG by 2.3-4.0dB, FRAC by 2.0-3.1dB).

We have further compared FWC and LZC with many other images such as Barbara, Baboon, Wood, Town and Lighthouse and detailed results are reported in [3]. They are not shown here due to space limitation. The actual gain of FWC depends on the coding image and the operating bit rate. Among 30 experiments, FWC is inferior to LZC only in one case by 0.02dB while, for all other cases, FWC outperforms LZC by 0.0dB to 0.8dB.

The FWC coded Lena is shown in Fig. 4(a), we marked the fractal prediction regions with $\oplus$ in Fig. 4(b). It has been observed that coupled with a state-of-art wavelet coder, approximately 8% of image regions in Lena can benefit from the use of fractal prediction. However, these regions are usually consuming a substantial amount of coding bits. For example, LZC devotes around 17% of the total coding bits

Table 1: Performance comparison for image Lena

Image Coder	Exp 1		Exp 2		Exp 3		Exp 4		Exp 5	
	Rate(bpp)	PSNR(dB)	Rate	PSNR	Rate	PSNR	Rate	PSNR	Rate	PSNR
JPEG	-	-	-	-	0.1855	28.61	0.3785	33.34	0.7643	36.68
FRAC	-	-	-	-	0.2175	30.71	0.4477	33.40	0.7626	35.92
PPC	-	-	0.18	31.2	0.26	32.78	0.37	34.0	-	-
EZW	0.0359	25.79	0.0825	28.66	0.1816	31.76	0.3694	34.93	0.7574	38.11
LZC	0.0359	26.33	0.0825	29.27	0.1816	32.51	0.3694	35.60	0.7574	38.63
FWC	0.0359	26.42	0.0825	29.41	0.1816	32.68	0.3694	35.84	0.7574	39.02

to the 8% fractal efficient regions in the Lena test image.

5. CONCLUSION

In this work, we propose a hybrid coder which adopts adaptive fractal prediction and wavelet residue coding. We use the wavelet coder as the primary coder and the fractal contractive mapping as an auxiliary tool which is applied only to regions with a positive rate saving. It is interesting to see that FWC outperforms LZC in nearly all coding experiments with a PSNR gain ranging from 0.0 to 0.8dB. This indicates that the incorporation of an additional fractal prediction unit in the wavelet coder does provide an improvement in the R-D performance at the expense of some additional computational complexity.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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(a)

(b)

Figure 4: Experimental results for the Lena image: (a) the proposed fractal wavelet scheme (0.3694bpp, 35.84dB), (b) regions efficient for fractal prediction (marked with $\oplus$).