Overlapping qubits

Rui Chao — USC
Ben W. Reichardt — USC
Chris Sutherland — USC
Thomas Vidick — Caltech

arXiv 1701.01062

Parallel self-testing of (tilted) EPR pairs via copies of (tilted) CHSH

Andrea W. Coladangelo
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The parallel-repeated magic square game is rigid

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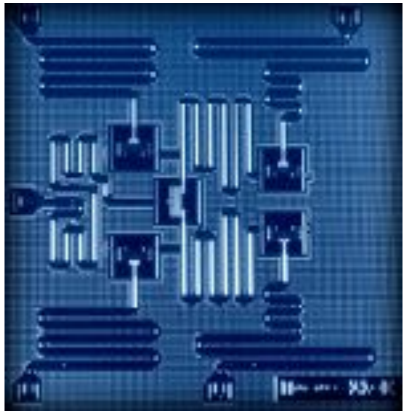
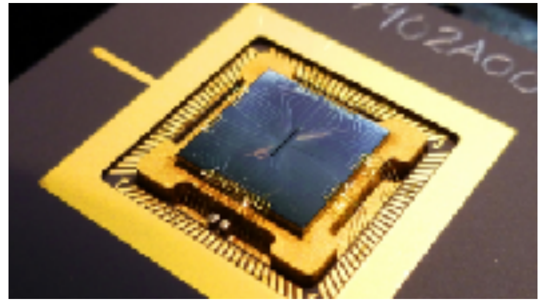
5 superconducting qubits, IBM



16 trapped ion qubits, UMD/NIST



1152 superconducting qubits, D-Wave

quantum computers are scaling up

n qubits ⇒ 2^n dimensions ⇒ exponentially hard to analyze

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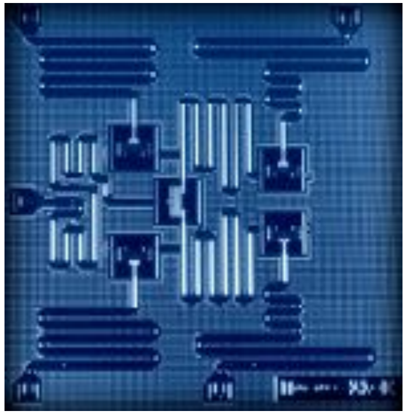

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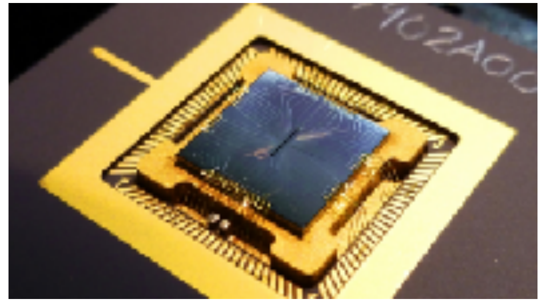

1152 superconducting qubits, D-Wave

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n qubits ⇒ 2^n dimensions ⇒ exponentially hard to analyze

How to test quantum computers?

small → *large*

state/process tomography

medium

error correction?

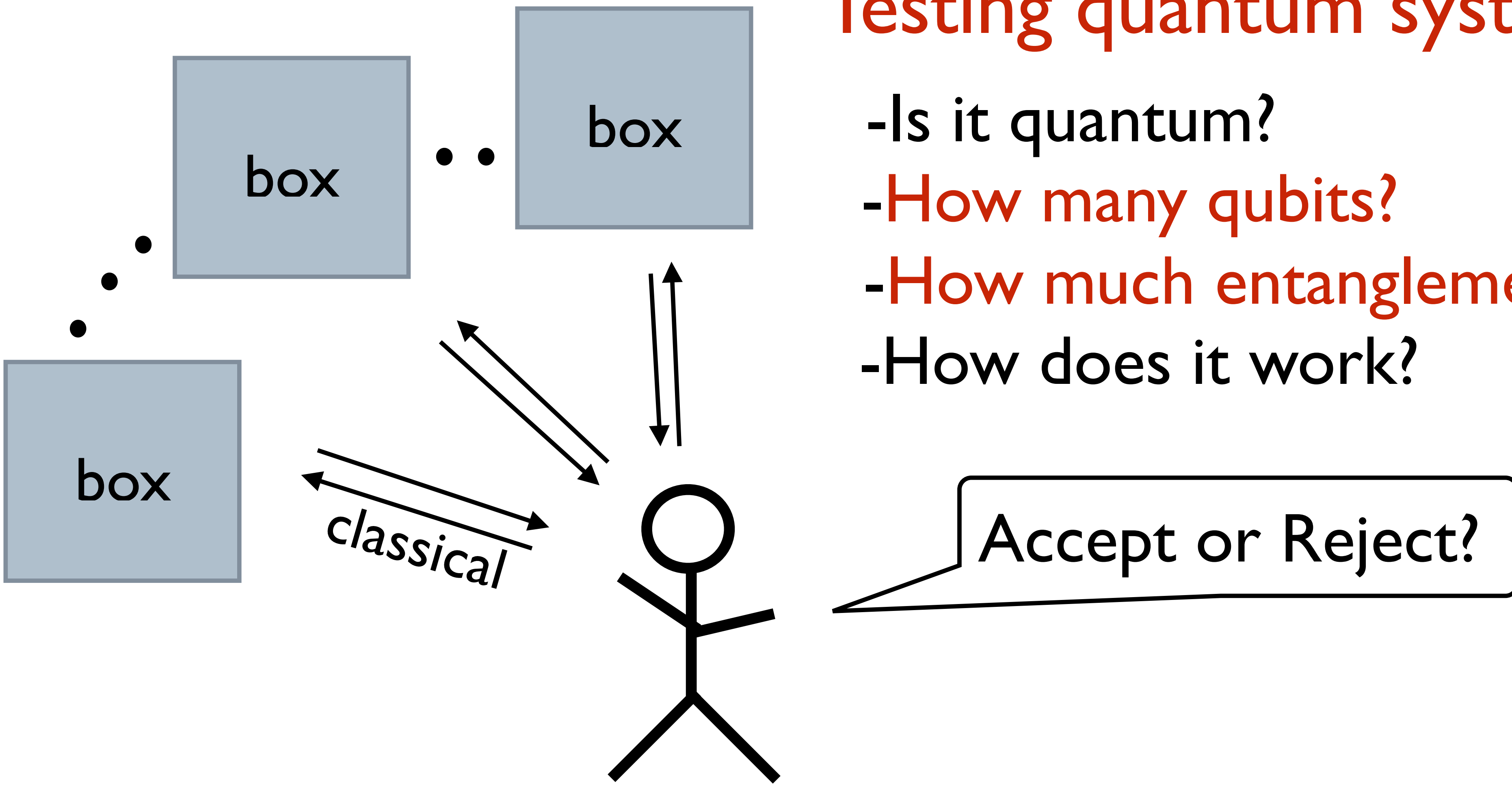
small simulation?

our tests !

factorization

Testing quantum systems

- -Is it quantum?
- -How many qubits?
- -How much entanglement?
- -How does it work?



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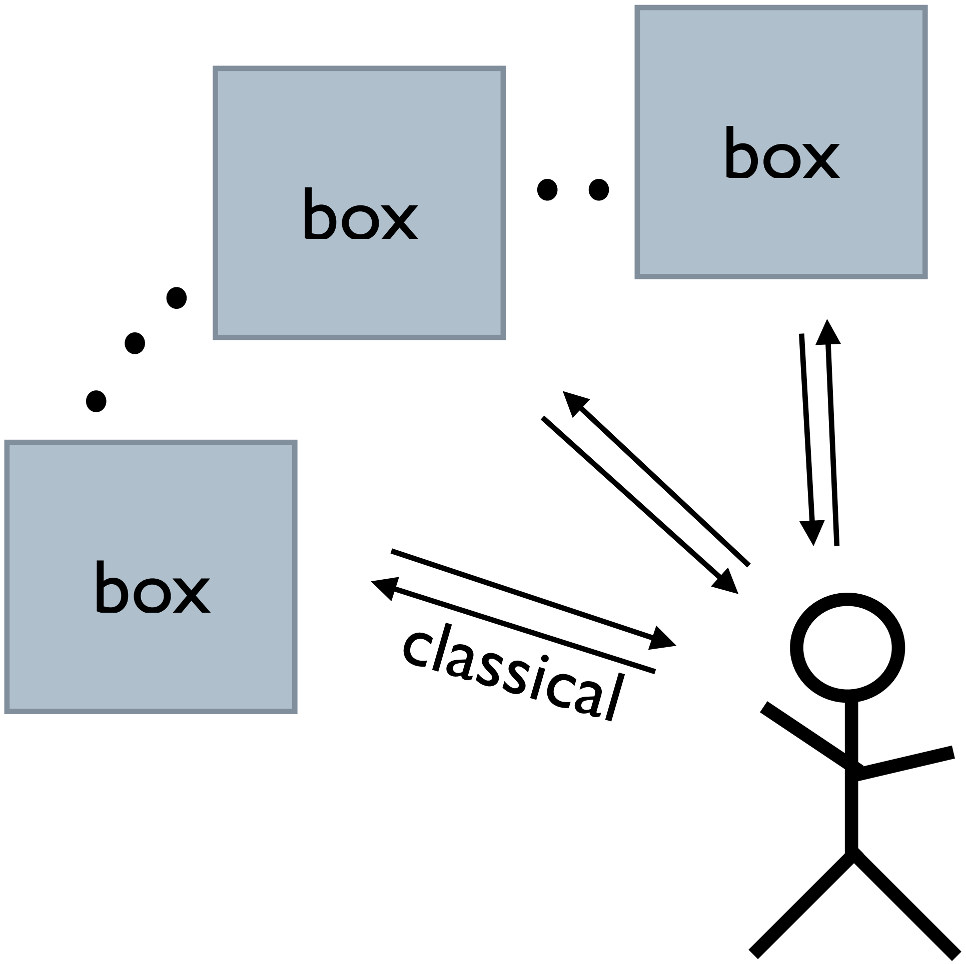


- -Is it quantum?
- -How many qubits?
- -How much entanglement?
- -How does it work?

Accept or Reject?

Goal: tests for large quantum systems — *scalability*

that take polynomial time — *efficiency*

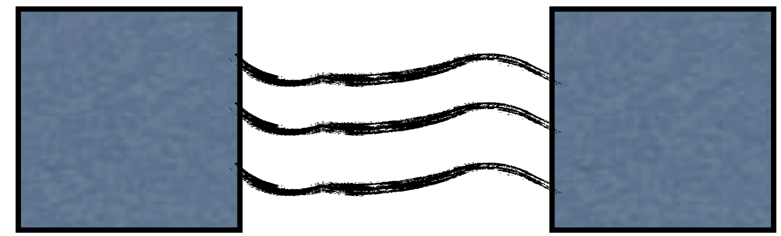
and(or) with high probability — *completeness & soundness*

and(or) tolerate constant noise — *robustness & rigidity* my part:

Test the dimensionality of a single quantum system

—How many qubits
^
overlapping

next:

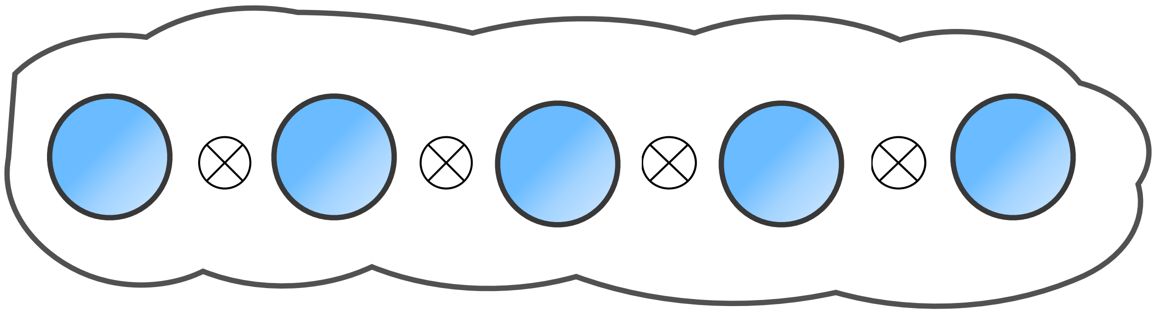


Test the number of (tilted) EPR pairs between two systems

—How much entanglement

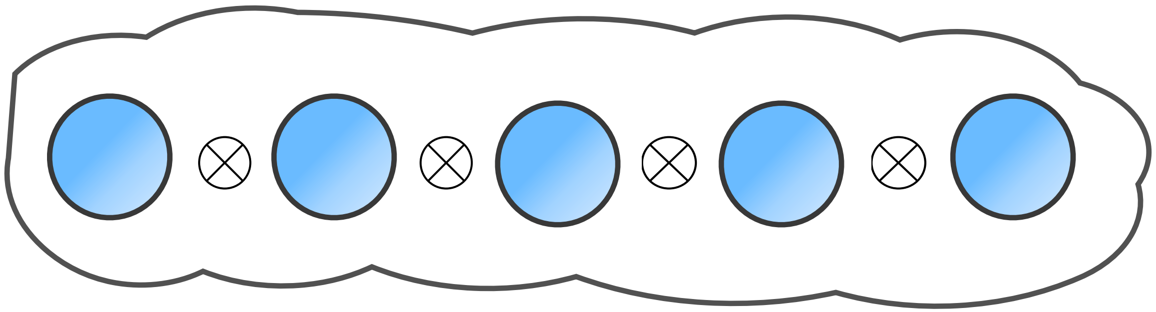
- Andrea:
  using tilted CHSH games
- Matthew:
  using Magic Square games

Quantum systems are made of qubits in tensor product



n qubits $\Rightarrow$ 2^n dim

Quantum systems are made of qubits in tensor product



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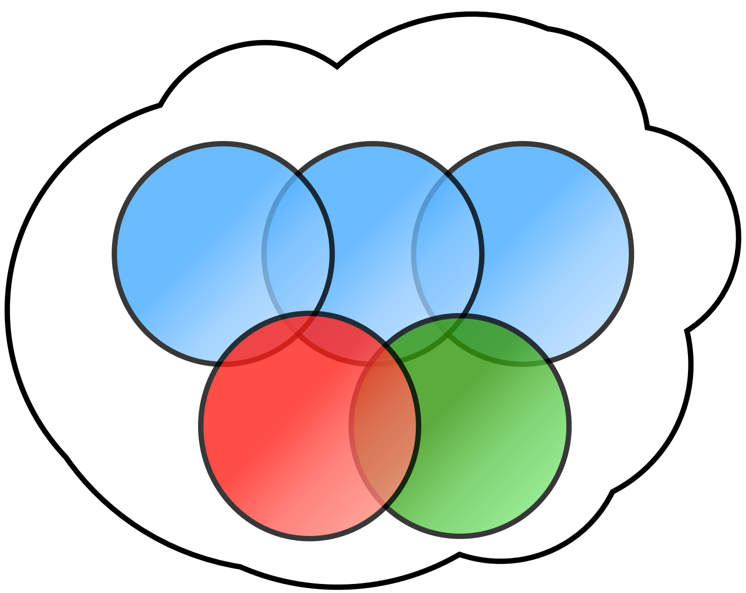
In general qubits can overlap



operations on one qubit
can slightly affect the others

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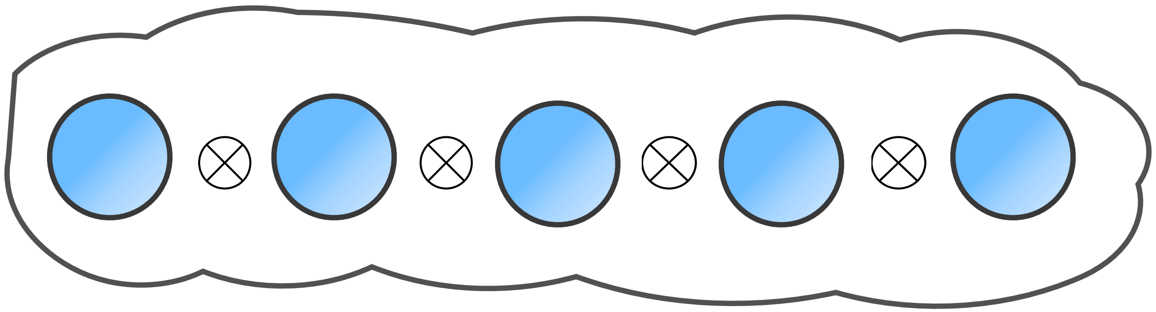

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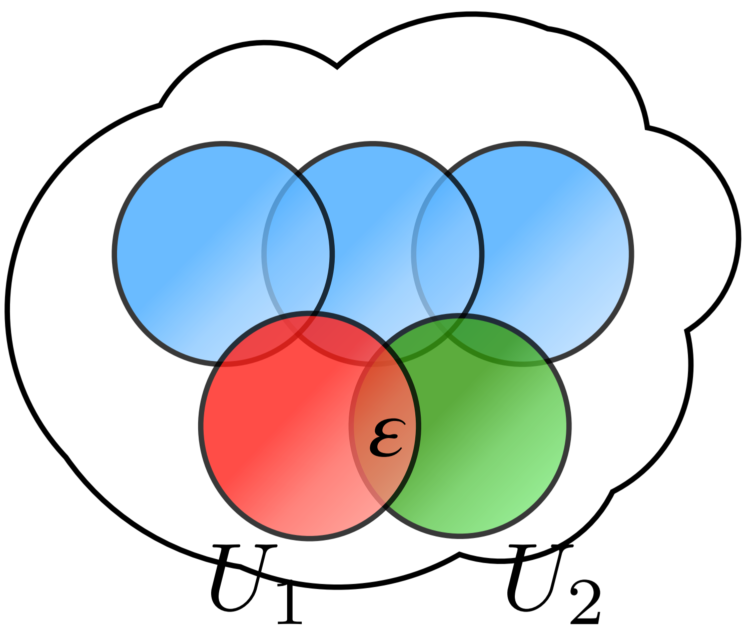

$$\|[U_1, U_2]\| \leq \epsilon$$

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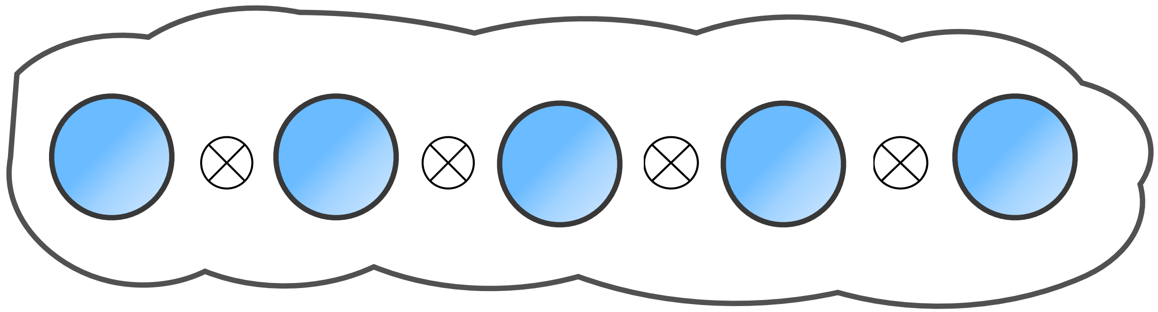

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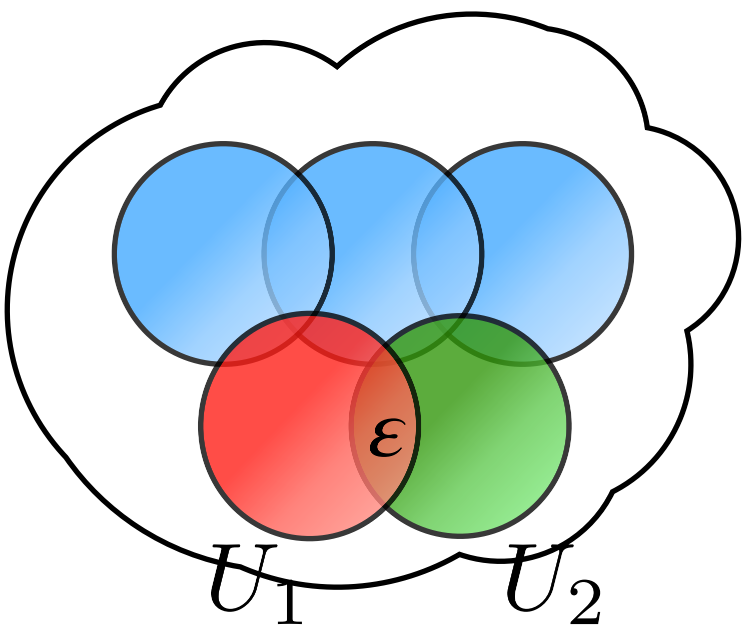

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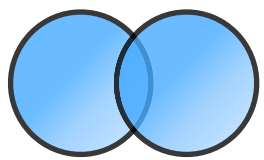
n ε-overlapping qubits

$\Rightarrow n^{1/\varepsilon^2}$ dim

Theorem 1:

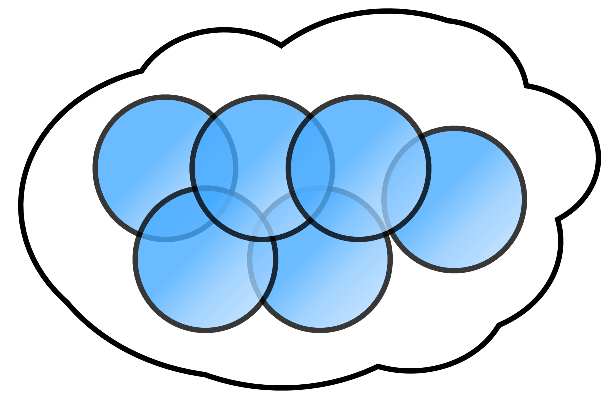
n overlapping qubits can fit in poly(n) dimensions



‘ε-overlap’

(operations on one qubit
can affect any other
qubit by at most ε)

$\Rightarrow$



n^{1/ε^2} dimensions

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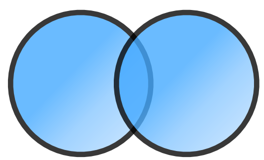

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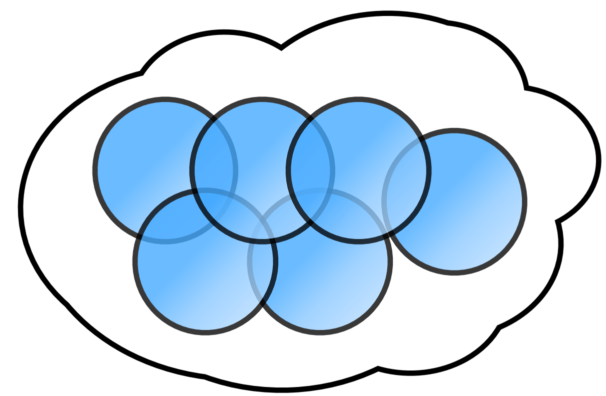

'ε-overlap'

(operations on one qubit can affect any other qubit by at most ε)

n^{1/ε^2} dimensions

Theorem 2:

Given access to n (overlapping) qubits, $\exists$ a test s.t.

$\Pr[\text{pass test}] \geq 1-\varepsilon \Rightarrow \text{dimension} \geq (1-O(n^2\varepsilon))\, 2^n$

Definitions:

- A qubit in $\mathcal{H}$ is a pair of anti-commuting reflections on it

$$\mathcal{H} \simeq \mathbb{C}^2 \otimes \mathcal{H}'$$

Indeed: $\{X, Z\} = 0 \Rightarrow$

$$X \simeq \sigma^x \otimes \mathbf{1}$$

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$\varepsilon=0 \Leftrightarrow$ qubits in tensor product:

$$X_1 \simeq \sigma^x \otimes I \otimes \mathbf{1} \qquad X_2 \simeq I \otimes \sigma^x \otimes \mathbf{1}$$

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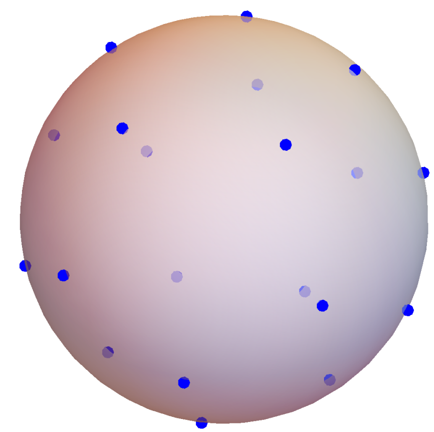
Theorem 1:

n ε-overlapping qubits can fit in $n^{\Omega(1/\varepsilon^2)}$-dimensional Hilbert space.

Proof idea:

nearly orthogonal vectors

$3n$ points in $\mathbb{R}^{O\left(\log n / \epsilon^2\right)}$



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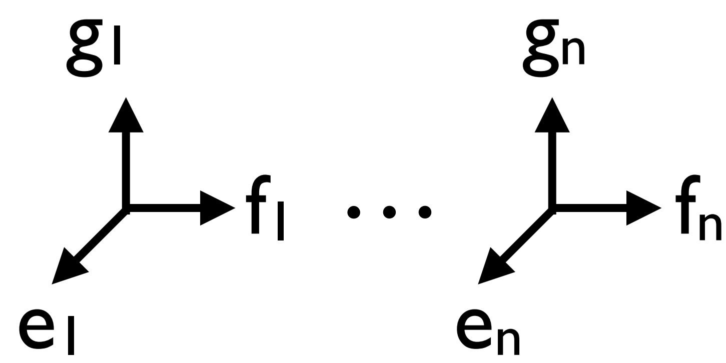

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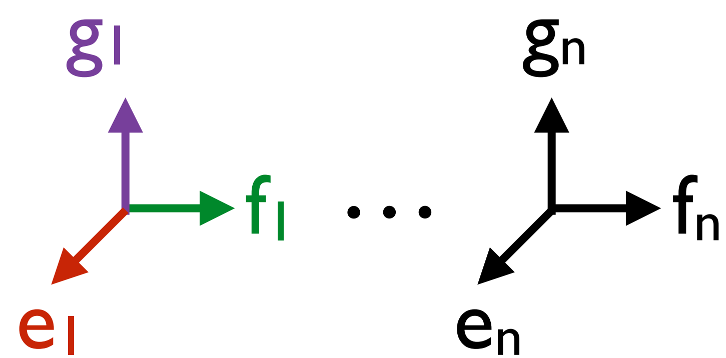

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($n^{\Omega(1/\varepsilon^2)}$-dim ref.)

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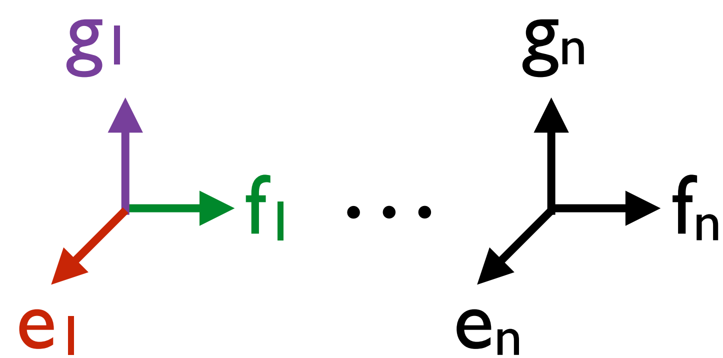

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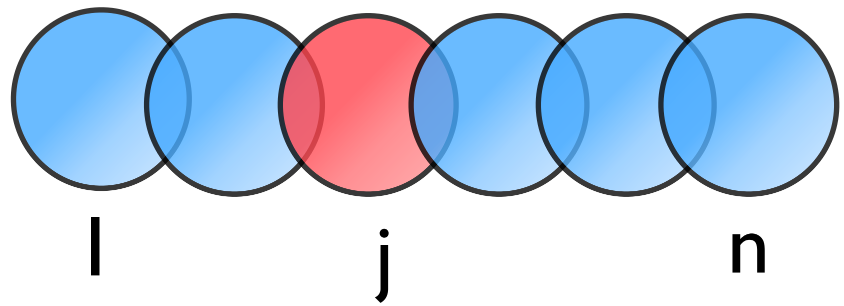
Note: meaningful only if $\varepsilon=\Omega(\sqrt{(\log n/n)})$

Dimension test: Given access to n qubits

1. Sequentially store n random qubits (|0›, |1›, |+›, or |-›)
2. Retrieve a random index & check it's correct



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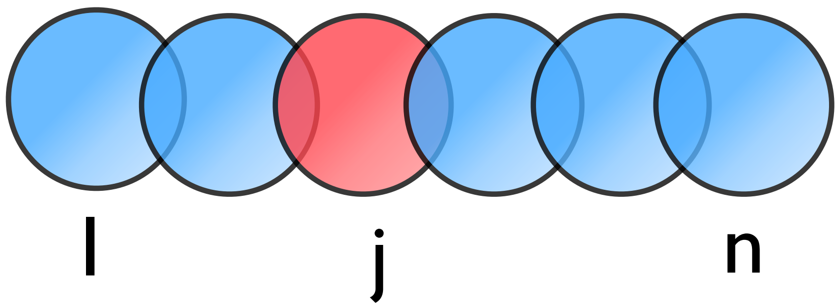

Theorem 2:

$$\Pr[\text{pass test}] \geq 1-\varepsilon \Rightarrow \text{dimension} \geq (1-O(n^2\varepsilon))\, 2^n$$

Note: meaningful only if $\varepsilon = O(1/n^2)$

Summary

- *Qubit:* anti-commuting reflection pair
- *Overlapping qubits:* nearly commuting reflections

- *Qubit packing:*
  n overlapping qubits can fit in poly(n) dimensions
- *Qubit separation:*
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- Test functionality
- Loosen assumptions & run experiments
- **Self-testing of EPR states**

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